Do Infants Possess a Rudimentary Understanding of Numbers and Mathematics?

Samara Doshi *

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Abstract

This paper describes math as enrichment to the modern mind, both in the forms of intellectual advancement as well as creative development. Not unlike language, math uses abstractions as a means of communication and allows a kind of "speaking without thinking". If mathematics truly is a kind of second language, it is worthwhile to understand the ways in which it develops within individuals and to examine at what age we actually begin to understand it. This paper reviews prominent views on these topics and concludes that, although infants possess a rudimentary understanding of mathematics, further research is needed to identify the exact age at which this understanding develops, and a study examining mathematical thinking among infants under the age of 4 months would be especially useful in advancing this area of inquiry.

1 Introduction

Numerical mathematics is a crucial tool used for both qualitative and quantitative analysis of several different fields including the modern sciences, the arts, as well as history [ea91]. This paper describes math as enrichment to the modern mind, both in the forms of intellectual advancement as well as creative development. There are a few eclectic ideas discussing the importance of mathematics and the significance it holds in our daily life. Lakoff and Núñez [Núñ00] take a more philosophical approach by describing math as more than just symbols and numbers. This paper follows that math is built from the ideas that transform these symbols into real world problems alongside their appropriate solutions. As such, math and cognitive sciences go hand in hand: Mathematics is not just a study; it is a philosophy that improves our critical thinking as well as problem solving skills ([Lak00]) Another paper by Wakefield [Wak00a] argues that math is like a second language. Harley [Har95] defines language as "a system of symbols and rules that enable communication." Following this definition,

*Advised by: Ms. Sori Baek of Princeton University and the Horizon Academic Research Program

math has no trouble qualifying as a second language. Wakefield theorizes that, like language, math is a skill that can be developed and improved upon with practice that also makes use of uniform symbols and rules. The paper states that, not unlike language, math uses abstractions, albeit in the form of numbers rather than words, as a means of communication. It also identifies the need for intuition and "speaking without thinking" as a similarity in both language and mathematics. This makes us question, if mathematics truly is a second language, is its development within individuals similar to that of language? If learning mathematics also requires intuition, at what age do we actually begin to understand it?

There are three distinct views on the age of numerical and arithmetic development (Cohen and Marks, 2002). Piaget [Pia41] followed a constructivist view, splitting development into four distinct stages: The sensorimotor stage from birth to 2 years of age. He theorized that, in this stage, children learn all things based on experience, or trial and error. They are incapable of anything more than basic motor movements. In stage two, or the preoperational stage, from 2 to 7 years of age, children develop language, memory and imagination. Their intelligence is intuitive, as they begin to make use of symbols of mathematics. The concrete operational stage from 7 to 11 years of age is when they actually begin to manipulate symbols and learn to apply and perform mathematics. The formal operational stage is the last one, indicating that the child can now apply abstract concepts of mathematics to the real world. Piaget was a firm believer of mathematics and the number concept developing only through prior experience with sensorimotor skills and intelligence.

In 1983, Kitcher argued against mathematical apriorism (i.e. mathematical knowledge gained independent of experiences), and supported the view of mathematical empiricism (i.e. mathematical knowledge gained through learning). Not unlike Piaget, he theorized the fact that children learn about numbers and mathematics mainly through learning and prior experience. However, he believed that this learning is facilitated by observations of numerical transformations and witnessing the possibilities and consistencies between events taking place, rather than sensorimotor skills and intelligence. He argued against the theory stating that the study of mathematics requires pure thought (i.e. knowledge not gained through observations). However, in recent years, a more nativist view has been developed by Wynn [Wyn92], who argued that infant sensitivity to numbers is innate, and could be portrayed through their understanding of complicated numerical situations from a very young age. She argues against both Kitcher and Piaget, and is a firm believer of mathematical apriorism.

Based on the opposing theories discussed above, this paper will serve to provide evidence for mathematical apriorism; through the discussion and meta-analysis of important studies, both new and old, this paper will highlight whether or not infants possess a rudimentary understanding of mathematics and numbers. By the end of this paper, the evidence provided would reject some of the above theories.

To help achieve the goal of discovering whether babies have a rudimentary understanding of mathematics, in this paper we will focus on the development of three distinct but interrelated concepts of mathematical abilities that are of interest to developmental researchers: discrete quantification (Lynn, 2004), continuous quantification [Mix02] and mathematics (Wynn, 1992). Discrete quantities refer to those that can only take certain fixed values (i.e. shoe sizes and the number of students in a class) as compared to continuous quantities that refer to those that can take any value (i.e. height and weight). This paper follows that, by analyzing infant understanding of discrete quantities, we are able to distinguish how individuals comprehend the arrays of smaller and larger numerical representations and how that is associated with the development of advanced cognitive functions. While discrete quantification discusses the representation of smaller and larger sets of numbers, continuous quantification involves the discrimination between volumes, areas and depths. Learning about how we develop a sense of continuous quantity may aid us in understanding the importance of the knowledge we have of the everyday interactions we have with our environment [Sim95] The application of mathematics involves an intricate number of cognitive skills that develop with practice and precision [But05]. It has also been determined that a stronger understanding of numbers (i.e. discrete quantities) and the concepts of volume and surface area (i.e. continuous quantities), builds the foundation required to have an extensive understanding of the study of mathematics [Cir16]. But do we gain all our mathematical knowledge through practice only as we grow older, or do we possess some perceptual and advanced abilities as infants that facilitate our comprehension of numbers and math? By what age can we actually understand and perform mathematics?

The question of whether or not infants understand numbers has sparked much controversy over the past few decades [Wak00b]. Researchers continue to conduct experiments and delve more into the depths of understanding infants' numerical capacities and the extent to which they can develop and grow [MW04] [MC06]. Through the course of this paper, we will discuss the significance of discrete and continuous quantification in human infants with the main aim of discovering whether or not they play a role in the development of mathematical abilities.

2 Discrete Quantity

As we have discussed previously, discrete quantity refers to those quantities that can only take definite, fixed values (i.e. how many). For the purpose of this paper, discrete quantification will refer to infants' understanding of arrays of numbers (i.e. a number sense). Initial evidence indicated that all humans have established a number sense (i.e. a sense of "how many") by the time they are approximately 5 years old [Fei04]. It is now well-accepted in the field that even 2-4 year olds have a basic understanding of number concepts [Sta80]. The ability to discriminate between small number arrays in 2-4 year olds has raised the question of whether younger infants possess this ability too. A study in 2005 [Xu03] has supported this claim by showing evidence that infants are capable of discriminating between number arrays and possess a "number sense".

In 1980, Starkey and Cooper conducted a violation-of-expectation experiment to show that infants are capable of discrete quantification based on their discrimination of small sets of visual arrays. The findings indicate that there is a precursor present, during infancy, before the onset of verbal counting in children. In this experiment, 5-month-old infants were initially habituated to arrays containing a set number of dots and then subsequently, in the post-habituation period, they were made to look at arrays containing a different number of dots. Researchers found that infants showed significant dishabituation when they were shown different numbers of dots, clearly indicating that 5-month-old infants were able to discriminate between small sets of arrays. However, the experiment also concluded that infants showed no dishabituation to the arrays containing a larger set of numbers indicating that infants are not able to discriminate between larger number arrays. This finding holds true even for 2-year-old infants who are able to perceive arrays containing up to three dots, but cannot perceive those with a larger number of dots [Mil83]. The researchers conducting this study showed evidence supporting the theory of infants being capable of discriminating between small number arrays (i.e. discrete quantities). Just the fact that infants of such a young age (5 months) could discriminate between various discrete quantities shows that they already possess a number sense and discrete quantification.

In tandem with this, another study in 2003 indicated that infants were capable of number discrimination from a very young age (Lipton and Spelke, 2003). The results show that 6-month-old infants were able to discriminate between 8 and 16 sets of sounds but not between 8 and 12 sets of sounds. This study shed light on the importance of the ratio between the number of sets of sounds played, as a discriminator of infant's ability to distinguish between discrete quantities. The results indicate that while 6-month-old infants have the ability to distinguish between sounds in a 2.0 ratio(i.e. 8 and 16 sets of sounds respectively), they lack this ability when the ratio is 1.5 (i.e. 8 and 12 sets of sounds respectively). When the same experiment was repeated with 9-month-old infants by the same researchers, it yielded shocking results: It was found that while they could discriminate between sets of sounds in a 2.0 ratio, they could also discriminate between sounds in a 1.5 ratio. However, when the ratio was smaller, once again, even 9-month-old infants were unable to distinguish between the sets of sounds. This study provides evidence for "number sense" or discrete quantification increasing with age, whilst still highlighting that this ability stems from a young age of 5-6 months.

Taking into account the study conducted by Lipton and Spelke (2003), an experiment was conducted by the same researchers in the same year, to discover whether the theory for infant discrimination of discrete quantities increasing with age held true for the same ratios but with even larger sets of number arrays. Since it had already been established that infants aged 6 months can distinguish between sets of arrays with a 2.0 ratio, this study repeated the same process with 16 and 32 sets of sound. The results showed that infants could successfully discriminate between the sounds, as measured by the violation-of-expectancy paradigm through infant looking time. This further supports the theory of infants' ability to distinguish between discrete quantities increasing with age, also proving that this theory holds true for much larger number arrays as well. Infants as young as 6 months old have the capability of distinguishing between most sets of numbers with a 2.0 ratio, regardless of how large the numbers are.

The aforementioned research shows that we develop a strong sense of numerical discrimination and discrete quantification right from infancy. The fact that infants have the ability to discriminate between sets of numbers in a 2.0 ratio even when the numbers are large (i.e. 16 and 32 sets of sounds) indicates that they possess advanced cognitive functions and numerical abilities from the age of 5-6 months (Lipton and Spelke, 2003). The evidence put forth by Lipton and Spelke as well as Gelman and Gallistel starkly contrasts the constructionist view put forth by Piaget (1952). If infants are capable of discriminating between discrete quantities and sets of numbers from the time they are 5 months old, it is evident that they don't have to be over 7 years of age to comprehend the concept of numbers or possess a "number sense" as Piaget suggested. While we can argue against Piaget's constructivist theories, it remains unclear whether the theories put forth by Wynn (1992) and Kitcher (1983) can be supported with evidence or not. The fact that infants are capable of performing advanced numerical functions like the discrimination between number arrays, could indicate that they gained this knowledge through experiences and observations of numerical transformations as Kitcher argued. However, it could also support Wynn's nativist view who argued that infant sensitivity to numbers is innate and independent of experience. However, regardless of which theory can be deemed most appropriate given the current evidence, it can be safely asserted that infant's from the age of 5 months already possess advanced cognitive and numerical functions which enables them to distinguish between sets of number arrays in a 2.0 ratio, regardless of how large the numbers are (Lipton and Spelke, 2003).

As our "number sense" continues to develop as we age, it is deep rooted within us from the time we are approximately 5 months old. While researchers don't tend to argue the theory of infants' understanding of discrete quantification, it has furthered their curiosity into discovering what else infants can be capable of. Although they possess the ability to distinguish between sets of numbers in a discrete quantity, are they able to further that ability and discriminate between continuous quantities too? Can they then integrate these concepts to actually comprehend mathematics from the age of 5-6 months?

3 Continuous quantity

While it is now established that infants possess an understanding of numbers, researchers continue to explore the ability of infants to distinguish between larger and smaller quantities as a whole, rather than just individual number arrays (i.e. continuous quantities) [Fei04]. Experiments have been developed to gauge spontaneous infant understanding of the concepts of "more and less." While studies on infant discrimination of discrete quantities only had infants distinguish between the number of items, studies delving into infants' understanding of continuous quantities (i.e. how much), record infant choice when factors such as volume, surface area and length are changed [Fei04]. If infants can interpret quantities and volumes of substances, their overarching cognitive framework may already be very developed, further supporting the ideas put forth by Wynn (1992) and Kitcher (1983) who argued that infants possess rudimentary mathematical abilities from a young age. If infants are incapable of discriminating between continuous sets of quantities, the constructionist theory put forth by Piaget may be the most accurate, as he believed that children gain these abilities very gradually as they grow older. By making the distinction of whether infants can discriminate between continuous quantities or not, researchers can be one step closer to interpreting how advanced the mathematical capabilities of infants truly are.

Some researchers argue that continuous quantification does not require attention to a certain number of individual arrays (e.g. dots, sounds) [Cle01] but rather can be measured by using overall amounts such as spatial extent (i.e. surface area) or contour length (i.e. the sum total of the perimeters of the individual objects in the set). Clearfield and Mix (1999) habituated two groups of infants to sets of two or three squares of the exact same size. In the post-habituation period, both groups saw a different display: One saw a familiar number of squares but a change in the spatial extent that was portrayed through a change in the overall contour length, while the other saw a different number of squares altogether while the contour length remained constant. The principal finding was that infants looked significantly longer at the stimulus with the changed contour length rather than the one with the different number of squares. This suggests that infants are more sensitive to a change in continuous quantity (spatial extent, area) rather than one in discrete quantity. Moreover, infants are sensitive and responsive to both: changes in discrete and continuous sets of arrays.

In a similar study conducted to confirm whether infants are more responsive to a change in continuous quantity over a change in number, 5-month-old infants were shown simple situations using dolls of different sizes [Fei04]. They were initially habituated to one large three-dimensional doll. In the post-habituation period, they measured infant looking time toward one small object or two small objects. In the second condition, they were habituated to two small objects and then shown displays with either one or two big objects. The results displayed that infants dishabituated to both conditions; they reacted to both a change in number (i.e. condition one) and a change in mass (i.e. condition two). However, infant looking time was much higher when the condition displayed a change in the mass of the doll. This further showcased infants' preferences in understanding of continuous quantities (i.e. mass) over discrete quantities (i.e. number). The fact that infants show sensitivity toward a change in the mass of objects from such a young age could indicate that they have a stronger understanding of numbers and mathematical concepts like volume, surface area and mass than researchers have previously accounted for. This, in turn, could imply that human infants do possess a rudimentary understanding of mathematics from such a young age.

While the experiment above conducted by Feigelson and Spelke indicates that infants show greater preference toward changes in continuous quantity over discrete quantity, there have been some questions raised regarding the accuracy of the experiment. It remains unclear whether variables such as contour length were taken into account while performing the experiment. Variables such as contour length are likely to vary with number unless they are explicitly controlled for [Kar69]. Therefore, the reason infants showed a preference toward a change in continuous quantity over discrete quantity could be because there was more than one variable being changed in the situation relating to a change in continuous quantity (i.e. mass and contour length could have both been altered).

In order to control for other continuous variables, and directly compare infant preference of continuous v/s discrete quantities, in 1995, Fagan and Miranda conducted an experiment comparing infant looking time toward two pairs of displays: one compared objects of different numbers with the same size and the other compared objects of different sizes but the same number. The overall mass of both displays was the same and the contour length was controlled for. The results yielded that infants preferred to look at the display comparing the continuous quantities (i.e. size) rather than the display comparing the number of items all of the same size (i.e. number). This confirms that infants display a direct preference toward changes in continuous quantities over discrete quantities. By displaying this preference, infants prove that they are capable of more complex cognitive functions and skills. This, in turn, makes them more likely to be able to perform mathematics as well.

In turn, these findings continue to support both Wynn's and Kitcher's independent views following that infants are capable of complicated cognitive functions from a young age, be it innate or through experiences and observations of numerical transformations. Once again, it seems as though all the research displayed above contradicts Piaget's constructivist theory following that children can only begin to comprehend mathematics and develop even the most basic "number sense" in the preoperational stage from 7-11 years of age.

It can now be asserted that, through an understanding of continuous quantities such as volume, contour length and mass, infants are one step closer toward comprehending mathematics. As previously mentioned, volume and surface area,amongst other things, are important concepts that build the foundation of the study of mathematics. By showing a direct preference toward more complicated change (i.e. volume over number) human infants are proving that their cognitive framework and capabilities are far more advanced than previously accounted for. This raises the question: if infants have an understanding of both continuous and discrete quantities and can identify changes in them, are they also able to apply these concepts to more complex mathematical manipulations in the real world?

4 Mathematics

While it can now be established that infants possess an understanding for both discrete and continuous sets of data, it is important to discuss how this ability impacts their mathematical skills as well. [Fra17] describes mathematics as the distinction between discrete and continuous data. This paper implies that the foundations of math rely on one's ability to correlate discrete and continuous data. Discrete mathematics involves concepts, techniques and skills that contrast those required by continuous mathematics. However, the ability to discriminate between the two "lies at the heart of mathematics."

As asserted in the previous sections of this paper, infants' perception of discrete and continuous quantities are factors that could impact their mathematical abilities as well. A study conducted by Butterworth in 2005, evaluated the evidence of infants' mathematical abilities rising from specific cognitive skills they developed through their understanding of numbers (i.e. their "number sense"). The researchers concluded that the foundations of mathematics depend upon one's understanding of numbers and the operations these numbers can fit into. General mathematical operations like addition, subtraction, multiplication and division can be defined in terms of sets of numbers or discrete quantities. However, more complex mathematical tools and techniques such as geometry and calculus require an understanding of continuous quantities such as length, mass and volume as stated by Franklin in 2017. As infant cognitive functions develop through the improvement in their ability to decipher changes in both discrete and continuous quantities, are they growing one step closer toward applying and performing mathematics?

In 1992, Wynn extended the line of investigation surrounding infants' comprehension of numbers and mathematics by performing experiments to discover their reactions to simple addition and subtraction problems. Initially, 5-month-old infants were familiarized to 2 pretrials: a presentation of one doll and then a presentation of two dolls, and then vice versa. Infants were then randomly assigned one of two situations: addition (1+1) or subtraction (1-1). Regardless of which situation they were assigned, they were shown two presentations. One presentation was of the right solution to the problem, and another was of an incorrect solution (i.e. 1-1=1 or 1+1=1). After the course of the study, Wynn reported that infants tended to look longer at the 'incorrect' solution rather than the 'correct' solution. She asserted that this was due to the violation-of-expectation paradigm that stated that infants typically look longer at situations that surprise them than those that meet their expectations. Based on these results, it can be established that infants possess a rudimentary understanding of simple mathematical problems manifested through a longer looking time when shown incorrect mathematical problems. Wynn argued that infants have an innate sense of numbers and mathematics, and can 'calculate the solutions of simple arithmetical operations on a small number of items.' If Wynn's theory were true and if infant understanding of mathematics truly is innate, then even younger infants (those below the age of 4 months) should be able to process and perform simple mathematical processes. While there are studies arguing that infants aged 4 months possess the ability to discriminate between sets of numbers in an increasing order, not enough studies have been conducted as of yet testing infants below the age of 4 months.

After Wynn's experiment, more studies began focusing on infant comprehension of mathematics [MW04] [MC06]. For example, in contrast to Wynn's innate and numerical approach, Simon [Sim95] argued that there could be a larger number of factors that influenced infant looking time in Wynn's experiments: one of such being memory. While both predicted that infants would look longer at the events that violated their expectancies, Simon argued that there were other possible reasons for this. Although infants had an understanding of the logic surrounding Wynn's experiments (i.e. when more input is added, the output should be more as well), they didn't necessarily understand the exact value of the output in a given situation. If this were true, then while infants had a rudimentary understanding of the concept of numbers, they may not possess the ability to calculate. However, refuting this claim, Wynn argued that since the infants had already been familiarized with the stimuli of both two and one doll presentations, and only one number was changed each trial (i.e either 1+1=2 or 2-1 =1), infants had to have possessed precision and understanding to be able to distinguish between sets of such small values. This shows that through their understanding of numbers and quantities, infants are capable of advanced and precise cognitive functions enabling them to distinguish between number arrays with tiny sets of values.

A similar meta analysis conducted to evaluate the reliability of Wynn's original findings revealed a high summary effect size of d = +0.34, supporting the theory of infants fixating toward 'mathematically incorrect' numbers of items, highlighting their rudimentary understanding of mathematics [Chr17].

Another study tested alternative explanations regarding the dishabituation of infants that looked longer at the unfamiliar solutions. These explanations tested the theory of how infants' understanding of objects and their behavior may be factors that could have affected their performance in earlier experiments [Koe97]. One such experiment examined the possibility that earlier results reflect the infant familiarization effect: Although when infants have been familiarized to certain stimuli they typically prefer novel stimuli in subsequent choice tests, there has been evidence of infant familiarity preferences (i.e. they prefer previously seen displays). In the early 2000's a few studies provided infants with varying levels of exposure to one or two visual arrays before showing them standard addition and subtraction events, with both correct and incorrect solutions [CM02]. These experiments used 8 familiarization trials in comparison to the 2 pre-trials conducted by Wynn, where infants were familiarized to only one or two objects. The results yielded no familiarization preferences for the 'correct' stimulus; instead the results supported Wynn's results, showing an increased looking time toward the incorrect stimuli. These experiments only further support Wynn's hypothesis and show evidence supporting infants' mathematical capabilities.

Therefore, while it remains unclear whether infants' understanding of mathematics is innate (as theorized by Wynn) or developed at a very young age through observations and experience (as theorized by Kitcher), it can be asserted that infants are capable of processing and understanding mathematics from as early on as the age of 4 months. However, despite a growing approval for the theories put forth by both Wynn and Kitcher, there remains a lot of debate regarding which of these two theories can be deemed more accurate [Cle06].

5 Discussion

Prior research suggests that infants possess an understanding of the concept of numbers or discrete quantity [Sta80]. They also display an ability to understand area, volume and length as a means to distinguish between the concepts of "more" and "less" [**?**]. Therefore, the fact that babies (5 months of age) can distinguish between small number arrays [Xu03], observe changes in quantities and understand simple mathematical operations (addition and subtraction) is a clear indicator of their mathematical capabilities. However, the overall extent and capacity of infants' mathematical abilities remains uncertain. Moreover, the age at which they develop this understanding remains unclear as well. While some researchers support the theory of infants' mathematical capabilities being present from birth [Wyn92], others believe that they gain this understanding through experiences, observations and other external factors [Cle01].

In this paper, we discussed three differing theories about the age at which children begin to understand mathematics: Piaget followed a constructionist view, splitting development into 4 distinct stages. He theorized that children only began to process and apply mathematics in stage three (i.e. the concrete operational stage), from the ages of 7-11. He theorized that children's application of mathematics was dependent on the sensorimotor skills they acquired in stage one of their development (i.e. the sensorimotor stage). Phillip Kitcher argued that although children understand mathematics through learning, this learning has nothing to do with their sensorimotor skills, it is built by their observations of the world around them. Both these theories imply that mathematics is not a skill that is present innately within children. It is developed through learning, observations and interactions with the environment. However, as discussed through this paper, there has been a myriad of evidence from many sources providing evidence justifying infants' mathematical capabilities [Cle01] [Wyn92]. Most of these papers support the theories put forth by both Wynn and Kitcher. While we know that infants aged 4 months and above possess the capabilities to perform advanced mathematical and cognitive reasoning, it remains ambiguous whether infants below the age of four months can also do the same.

Therefore, it can be concluded that, although infants possess a rudimentary understanding of mathematics, the exact age at which this understanding develops can only be asserted after further research is conducted on infants below the age of 4 months' abilities to comprehend mathematics.

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