A Mixed-Integer Linear Programming Approach to Optimize Tennis Regimens for Young Athletes in Korea

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ABSTRACT

The popularity of tennis has been surging among young individuals in South Korea, as it offers an appealing alternative to other sports like golf. With this surge in popularity, there is a corresponding increase in competitive intensity, making the optimization of training regimens crucial for success. While athletes in this sport commonly engage in a variety of training activities there has been limited systematic research that offers a quantitatively validated approach for balancing these activities to maximize on-court performance. This study aims to fill this gap by providing an optimization model tailored to the specific requirements of tennis training. Through surveying 48 prospective athletes in South Korea, the study establishes upper and lower bounds for each of 4 activities: Serving Practice, Ground Strokes Practice, Physical Conditioning, and Mental Training. Multiple regression analysis is then used to calculate the weights of each training activity in an objective function, formulated to measure on-court performance. A Mixed-Integer Linear Programming (MILP) model is employed to allocate optimal weekly training hours to each activity, based on these weights and constraints. The study concludes by identifying an optimal training schedule that maximizes the value of the objective function, thereby optimizing on-court performance. The convergence of the objective and best bound at a low level further substantiates the optimality of the discovered solution.

Introduction

Designing training programs for tennis is a challenging task owing to many physiological variables that are key to optimizing performance. Tennis is characterized by a high degree of unpredictability (Kovacs, 2006), affecting various aspects of gameplay. Factors such as point length, shot selection, and overall strategy can change rapidly within the context of a single match, requiring players to adapt in real-time. External variables, such as weather conditions and the specific strategies employed by the opponent, further add layers of complexity. These uncertainties impose a dynamic range of physiological demands on the player, including fluctuations in aerobic and anaerobic energy systems, muscle activation patterns, and cardiovascular responses. Consequently, the capacity to adapt to a volatile environment is essential for success in this sport.

Strokes are generally classified into three main types: groundstrokes, volleys, and overhead strokes (Nugroho *et al.*, 2023). Groundstrokes are particularly crucial for establishing control and dictating the pace of the game. This technique involves swinging a tennis racket to generate a forceful yet controllable impact against a ball that has rebounded from the ground. The objective is to produce a shot with a high degree of accuracy, targeting a specific area of the opponent's court. Groundstrokes can be executed either through a forehand or a backhand swing, depending on the player's position relative to the incoming ball.

The execution of groundstrokes necessitates a high level of coordination, particularly in footwork and swing mechanics. Proper footwork is essential for positioning oneself optimally to make the stroke, while the swing mechanics are vital for generating the desired force and direction of the ball. The coordination between these two elements is often the distinguishing factor between amateur and professional players. Inadequate footwork can result in poor positioning, which subsequently affects the quality of the swing, leading to less accurate and less powerful shots.

(Périard *et al.*, 2014) argued that tennis demands not only physical agility but also a high level of cognitive functioning. One of the key mental requirements of the game is the capacity for anticipation, which involves predicting the opponent's next move based on a range of cues such as body posture, the angle of the racket, and the trajectory of the ball. This enables a player to position themselves optimally and prepare for the next stroke. In addition to anticipation, reaction time is another crucial mental faculty required in tennis. Upon recognizing the direction and speed of an incoming ball, a player must instantaneously decide the most effective counter-stroke and execute it with precision. These split-second decisions are often the determining factor in point outcomes, especially during high-stakes moments of a match.

Tennis matches can last for several hours, requiring sustained energy and focus. Conditioning exercises aimed at improving cardiovascular fitness can significantly enhance a player's endurance, allowing them to maintain a high level of performance throughout the duration of a match. Moreover, physical conditioning is not just about enhancing performance but also about injury prevention. Tennis players are prone to a range of injuries including, but not limited to, tennis elbow, shoulder injuries, and leg strains. A well-rounded physical conditioning program that includes elements of strength training, flexibility exercises, and proprioceptive training can significantly reduce the risk of these injuries. Proper conditioning ensures that the musculoskeletal system is prepared for the stresses and strains of competitive tennis, thereby allowing players to maintain peak performance throughout a match and a season. It also enables quicker recovery post-match, which is vital in tournaments where players may be required to play multiple matches in a short period.

(Perkins and Davis, 2006) showed that with a lower level of physical conditioning, adolescent tennis players are particularly susceptible to overuse injuries, most commonly manifesting as muscle and ligament strains or sprains. The lack of initial physical preparedness makes this demographic more vulnerable, as their musculoskeletal systems may not be adequately conditioned to handle the repetitive and high-impact nature of tennis activities. This situation is exacerbated by the fact that adolescents are often in a phase of rapid growth and development, which can further contribute to biomechanical imbalances and increase the risk of injury.

In the early stages of long-term athlete development, specifically targeting young, there is a significant emphasis on the mastery of tennis-specific skills (Reid *et al.*, 2007). Training regimens for these young athletes are often rigorous, with technical and tactical sessions frequently exceeding 15 to 20 hours per week (Fernandez-Fernandez *et al.*, 2020). The rationale behind this intensive training schedule is to build a strong foundational skill set that will serve the athlete well as they progress to higher levels of competition.

Given the high training demands for young athletes the optimization of training regimens becomes a critical factor in their development and future success. The risk of overuse injuries, as highlighted by (Perkins and Davis, 2006), further emphasize the need for a balanced and scientifically grounded training regimen. This is particularly relevant for prospective athletes in South Korea, where the sport is gaining traction and the competition is intensifying. The increasing popularity of tennis in South Korea is evidenced by the upward trend in search queries related to the sport, particularly starting from early 2022 (Woo-hyun, 2022). This surge in interest spans across various age groups, indicating a broadening appeal of tennis as a recreational and competitive activity. The study's focus on South Korea offers a unique perspective, given the rising popularity of tennis in the region. While the sport has been well-studied in Western contexts, less attention has been given to tennis training regimens athletes in South Korea. The application of Mixed-Integer Linear Programming (MILP) in this study aims to address these concerns by providing a quantitatively validated framework for optimizing training schedules.

Method

Mixed-Integer Linear Programming (MILP) is a specialized subset of optimization algorithms that generalizes Linear Programming (LP) by permitting a portion of the decision variables to assume discrete integer values. In traditional LP models, the objective function and all constraints are represented as linear equations or inequalities, and the decision variables can take continuous values within a feasible region. In contrast, MILP imposes an additional layer of complexity by mandating that certain variables adopt only integer values, either within a specified range or as binary variables (Doganis and Sarimveis, 2008; Jünger *et al.*, 2009; Pirim and Aghalari, 2022). This integration of integer constraints makes MILP suitable for capturing intricate real-world phenomena that have both continuous and discrete components. The mathematical formulation of a MILP problem usually comprises a linear objective function to be maximized or minimized, subject to a set of linear constraints that define the feasible region, and integrality constraints on specified decision variables.

The advent of MILP has had a profound impact on various industrial and commercial sectors by providing a rigorous mathematical framework for tackling complex optimization problems. In manufacturing, MILP is instrumental in production planning, inventory management, and workforce scheduling, allowing for the realization of efficient and cost-effective operations (Tawarmalani and Sahinidis, 2002; Nowak, 2005; Plà-Aragonés, Rodríguez-Sánchez and Rebillas-Loredo, 2013; Belmonte, Aviso and Benjamin, 2022; Sporleder, Rath and Jansen, 2023). In the financial industry, MILP techniques are employed for portfolio optimization, where the goal is to allocate assets in such a way as to maximize returns while adhering to various risk constraints. The utility of MILP extends to the healthcare sector as well, where it aids in resource allocation, staff scheduling, and patient routing. Transportation logistics frequently utilizes MILP for vehicle routing, fleet management, and scheduling, aiming to minimize fuel consumption and transit times while complying with various regulatory requirements.

Solution techniques for Mixed-Integer Linear Programming (MILP) problems are generally more complex than those for Linear Programming (LP) due to the introduction of integer constraints, which create a non-convex feasible region. Various algorithms have been developed to find optimal or near-optimal solutions to MILP problems. Among the most commonly employed are the Branch-and-Bound Method, Cutting Planes, and Heuristic Methods (Aardal and Others, 2005; Khor, 2021).

The Branch-and-Bound Method is a widely utilized algorithmic approach for solving MILP problems. It is an exhaustive search procedure that systematically partitions the feasible region into smaller subregions and explores these to find the optimal solution (Barth and Bockmayr, 1995; Eckstein, 1996). The method employs a tree-like data structure where each node represents a subproblem. At each node, a relaxation of the MILP problem is solved to obtain an upper or lower bound on the objective function. These bounds are then used to prune branches of the tree that cannot contain an optimal solution, thereby reducing the search space. The algorithm proceeds by branching and bounding until it either finds an optimal solution or confirms that none exists within specified tolerances. Although the method is guaranteed to find a globally optimal solution, it can be computationally intensive for large-scale problems.

Cutting Plane methods, on the other hand, are iterative techniques that refine the feasible region by adding additional linear constraints, known as "cuts," to eliminate non-optimal integer solutions (Barth and Bockmayr, 1995; Aardal and Others, 2005). Initially, a relaxation of the original MILP problem is solved. If the solution contains non-integer values for variables that are supposed to be integer, a cutting plane is generated to exclude the current fractional solution. The LP problem is resolved with the additional cut, and the process is repeated until an integer solution is found. This method is particularly useful when the feasible region contains many fractional vertices and can often be combined with Branch-and-Bound to improve computational efficiency.

Heuristic Methods offer a different approach by prioritizing computational speed over optimality. These methods aim to find good, if not optimal, solutions in a relatively short period. Algorithms such as simulated annealing, genetic algorithms, and tabu search fall under this category. While they do not provide the same guarantees of optimality that exact methods like Branch-and-Bound and Cutting Planes offer, heuristic methods have the advantage of being faster and more scalable (Parker and Rardin, 1982; Holmberg, 1998). They are often employed as a first step in solving large-scale MILP problems to provide a reasonable initial solution, which can then be refined using exact methods.

In its standard form, a MILP can be described using linear equations and inequalities. The objective function is to maximize or minimize a linear expression subject to linear equality and inequality constraints. Mathematically, this can be represented as Equation 1 as follows:

Equation 1:

$$\text{Maximize or minimize } Z = c_1x_1 + c_2x_2 + \cdots + c_nx_n$$

Subject to:

$$\begin{aligned}
&a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \leq b_1\\
&a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \leq b_2\\
&\vdots\\
&a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \leq b_m\\
&x_i \geq 0, i = 1,2,\cdots,n\\
&x_j \in \mathbb{Z}, j \in \{\text{ set of integer variables }\}
\end{aligned}$$

The constraints or inequalities restrict the feasible region for the variables. These constraints are often based on real-world limitations such as resource availability or production capacities. The objective function Z, which is to be maximized or minimized, reflects the goal of the optimization problem. In a typical MILP problem, decision variables can be both continuous and integer. The continuous variables x_i can take any real value within the specified bounds, while the integer variables must take on integer values within their specified limits.

The Bounded Knapsack Problem (BKP) is a variant of the classic Knapsack Problem, a major problem in mathematical optimization. This problem can be framed within the paradigm of Mixed-Integer Linear Programming (MILP) due to its inherent discrete nature and its reliance on linear constraints and objective functions. The Bounded Knapsack Problem seeks to maximize the total value of items placed into a knapsack, subject to a weight constraint, with the added complexity that each item type has a limited quantity available.

In mathematical terms, the BKP is characterized by a linear objective function that aims to maximize the total value of the selected items. Each item i in the knapsack contributes a value v_i and has a weight w_i. The knapsack has a maximum weight capacity W. Unlike the classic Knapsack Problem, where an unlimited number of each item type can be included, the Bounded Knapsack Problem restricts the number of each item type to a maximum of b_i, introducing a layer of complexity to the decisionmaking process. The problem can be formulated as equation 2:

Equation 2:

$$\text{Maximize } Z = \sum_{i=1}^{n} v_i x_i$$

Subject to:

1. $\sum_{i=1}^{n} w_i x_i \leq W$ (Weight Constraint)
2. $0 \leq x_i \leq b_i, x_i \in \mathbb{Z}$ for all i (Boundedness and Integrality Constraints)

where x_i represents the number of items of type i chosen, v_i is the value of item i, w_i is the weight of item i, W is the maximum weight capacity of the knapsack, and b_i is the maximum available quantity of item i.

The BKP has significant practical applications, such as those of MILP, in fields such as resource allocation, budget management, and logistics. For example, in project management, the BKP can be used to select the most valuable projects to undertake within a fixed budget constraint, with each project having limited availability in terms of resources or time slots.

The solution approaches for the Bounded Knapsack Problem within the MILP framework typically involve methods like Branch-and-Bound, Dynamic Programming, and Greedy Algorithms, each with its own set of trade-offs between computational efficiency and solution optimality.

Branch-and-Bound, for instance, remains a major technique for MILP problems and is well-suited for BKP as it can effectively navigate the discrete decision space by systematically exploring and eliminating suboptimal branches, thus converging on an optimal or near-optimal solution. Dynamic Programming, another powerful approach, breaks down the BKP into simpler subproblems and solves them recursively, making it particularly effective for problems with overlapping subproblems and optimal substructure. Greedy Algorithms, while not always guaranteeing optimal solutions, can provide fast and often satisfactory solutions, particularly useful in scenarios requiring quick decision-making or as a starting point for more complex algorithms.

Problem Formulation

Serving is often cited as the most important shot in the game due to its role in initiating each point and providing the server with an opportunity to secure an immediate advantage. To excel in this critical aspect of the game, players employ a variety of strategies, from focusing on accuracy and speed to mastering diverse types of serves such as flat serves, slice serves, and kick serves. Similarly essential to a comprehensive tennis training regimen are the ground strokes, which encompass both forehand and backhand techniques vital for effective rallying and point-winning during match play. Within a typical ground strokes practice session, various drills are conducted with the aim of improving essential aspects such as ball control, depth, and spin. Drills commonly include cross-court and down-the-line exercises to simulate conditions that players will encounter in actual matches. To ensure consistent and challenging practice, ball machines and hitting partners are commonly utilized, and increasingly, technological aids are integrated for high-level performance analytics.

Physical conditioning can be regarded as foundational layer upon which all tennis-related skills are built, especially given the sport's high demands for strength, speed, agility, and endurance. A rigorous and structured physical conditioning program usually features a balanced mix of aerobic exercises, such as running and cycling, and anaerobic activities like sprinting and plyometrics. Strength training that targets both upper and lower body muscles is integrated into the regimen, along with flexibility and balance exercises aimed at enhancing range of motion and preventing injuries.

The sport requires a blend of concentration, emotional control, and strategic thinking, all of which can be nurtured through a variety of psychological techniques such as visualization, positive self-talk, and mindfulness. To further improve these skills, many players engage the services of sports psychologists who help them develop strategies for handling the psychological stressors that inevitably arise during competitive matches, such as managing pressure points and recovering from setbacks.

In the optimization problem of our study, the following decision variables were chosen to represent the hours dedicated to Serving Practice, Ground Strokes Practice, Physical Conditioning, and Mental Training respectively in a week. As motioned before, These variables were selected because they encapsulate the core components of a tennis training regimen, each with its distinct impact on overall performance. The following formula, equation 3, calculates the performance of the athletes.

Equation 3:

$$MW = \sqrt{\frac{\sum (pw - pl)^2}{SA}}$$

where,

1. Matches Won (MW): This component directly relates to the player's ability to clinch victories. The more matches a player wins, the higher the performance score. It serves as the primary multiplier in the formula.
2. Points Differential: It calculates the squared differential between Points Won (pw) and lost (pl) in each set. Squaring this differential emphasizes the importance of a clear margin of victory or defeat within individual sets, rewarding dominant performance and penalizing inconsistent performance.
3. Serve Accuracy (SA): The denominator, serve accuracy %serve accuracy %, serves as a correction factor. A higher serve accuracy would lead to a smaller divisor, hence amplifying the score. A lower serve accuracy would decrease the score. This component reflects the importance of the serve in tennis, as it starts every point and can be a critical determinant of a player's dominance in a match.

We made the number of matches won a primary multiplier in order to prioritizes the most basic and direct measure of a player's success, that is, their ability to win matches. Again, by squaring the points differential, the formula places a significant emphasis on not just winning, but winning with a clear margin. This can differentiate between players who barely clinch victories and those who dominate their opponents. Serve is a fundamental aspect of tennis. A player's ability to serve accurately can be indicative of their skill level, confidence, and potential to dictate the course of a match.

Decision Variables

1. (x_S) : Hours dedicated to Serving Practice (per week)
2. (x_G): Hours dedicated to Ground Strokes Practice (per week)
3. (x_P) : Hours dedicated to Physical Conditioning (per week)
4. (x_M) : Hours dedicated to Mental Training (per week)

Deriving Objective Function

This study applied a data-driven framework to formulate the objective function for the MILP. 48 prospective athletes in South Korea were surveyed to establish upper and lower bounds for each of the aforementioned training activities: Serving Practice, Ground Strokes Practice, Physical Conditioning, and Mental Training. These bounds serve to define the feasible region within which the decision variables can operate, reflecting realistic constraints such as time availability, physical stamina, and other logistical considerations. Subsequently, multiple regression analysis was employed to calculate the weights of each training activity, effectively quantifying their respective contributions to on-court performance. The residual-performances of the model were shown in figure 1, indicating a good-fit model. These weights are integral components of an objective function formulated for the optimization problem, aimed at maximizing on-court performance within the established constraints. The following objective function was derived:

$$Z = 3x_S + 5x_G + 4x_P + 2.5x_M$$

Where, we want to maximize Z

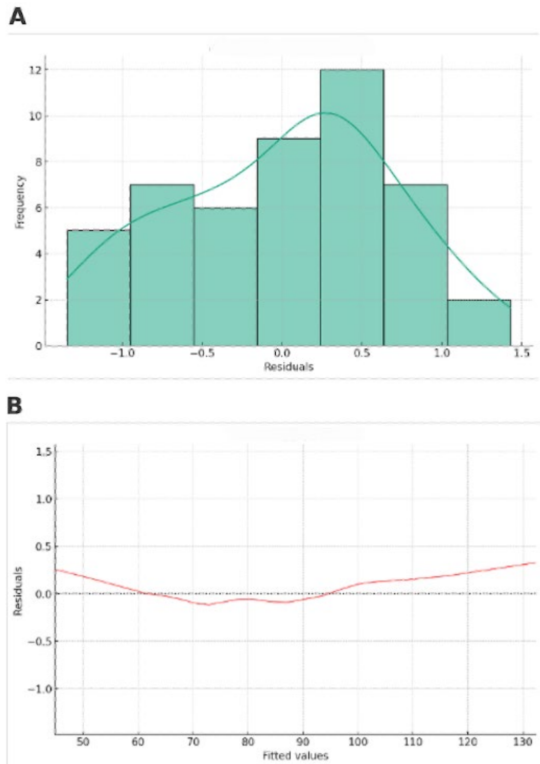

Figure 1. The performances of the derived objective function. As seen in figure 1A, the Histogram of Residuals is shown. The residuals are mostly centered around zero. As seen in figure 1B, the plot of Residuals vs Fitted is shown, which indicates a good-fit.

Constraints

The survey conducted with 48 prospective athletes in South Korea provided empirical data to establish time limitations for four core training activities: Serving Practice, Ground Strokes Practice, Physical Conditioning, and Mental Training. The total number of training hours that could be allocated to these activities in a single week was restricted to no more than 13 hours per week. This calculation was doe by averaging the responses of 5 trainers. This limitation accounts for the physical and mental demands of intensive training and aims to reduce the risk of overtraining and injury. It also allows for essential non-training activities, such as rest, nutrition, and strategy discussions, which, although not part of the four key activities under consideration, are nonetheless critical to an athlete's overall performance and well-being.

For each specific activity, this study set minimum and maximum time limits based on empirical data and practical reasoning. For example, Serving Practice was restricted to a narrow time frame to minimize the risk of injuries that can result from overusing certain muscle groups, particularly those around the shoulder and elbow. On the other hand, Ground Strokes Practice was allocated a longer time frame due to the diversity of movements involved, which distributes physical strain more evenly across multiple muscle groups. Physical Conditioning was given a flexible range to allow for the integration of different types of exercise, targeting various aspects of physical fitness. Mental Training was given at a shorter time frame, as the survey indicated that prolonged mental training sessions could lead to mental fatigue, thereby diminishing the effectiveness of this essential aspect of preparation.

Constraints, hours per week:

1. Total training hours (per week): $(x_S + x_G + x_P + x_M \leq 13)$

2. Serving practice: $(2 \leq x_S \leq 3)$
3. Ground strokes practice: $(4 \leq x_G \leq 6)$
4. Physical conditioning: $(2 \leq x_P \leq 4)$
5. Mental training: $(1 \leq x_M \leq 2)$

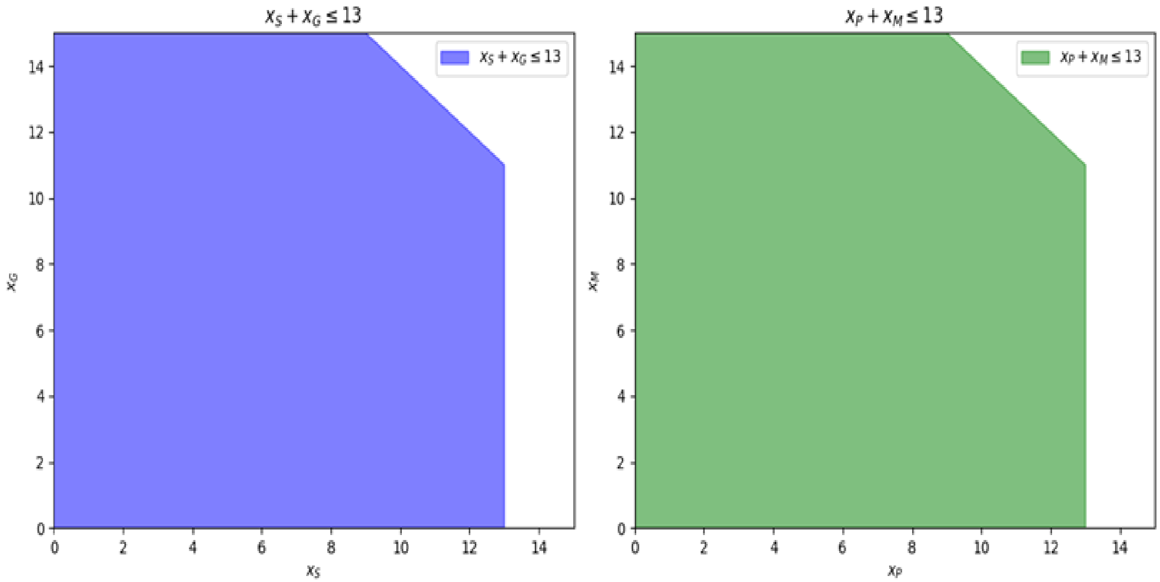

Figure 2. Plot for training hour constraint. Assuming (right) M_p=0 and Xm=0; (left) assuming Xs=0, and X_G=0

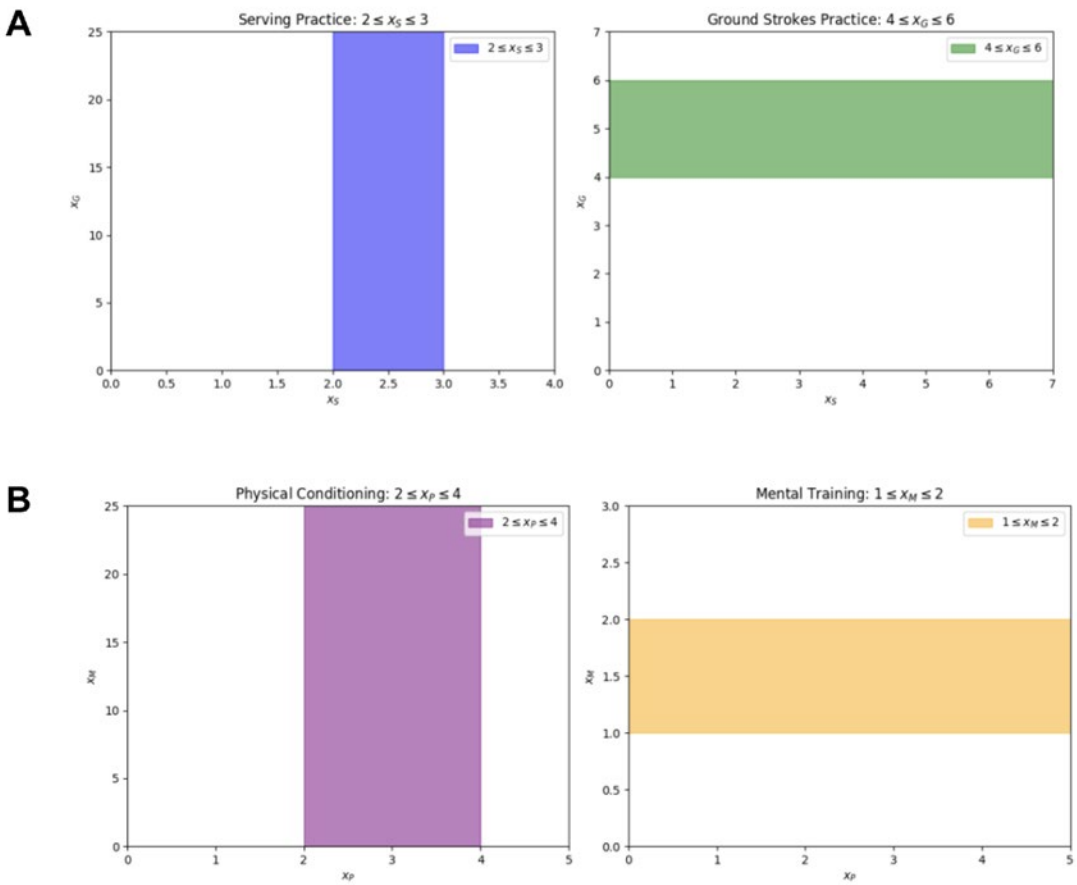

Figure 3. As seen in plot 3A, constraints plots for Serving practice and Ground strokes practice. As seen in plot 3B, constraints plots for Physical conditioning and Mental training.

Integer and Continuity Constraints

For activities such as serving practice, ground strokes practice, and physical conditioning, it is typical for tennis players to allocate whole hours or set time blocks. For instance, serving and ground stroke drills might be organized in specific sequences or repetitions that fit neatly into hour-long sessions, making fractional hours impractical or less effective. In the case of physical conditioning, exercises might be planned in circuits or sets which are also structured to last an entire hour, optimizing the rest and work period. Using integers ensures that the player fully engages in the whole routine without breaking the flow or consistency which could potentially impact the effectiveness of the training.

On the other hand, mental training, which might include activities such as visualization, meditation, or even psychological counseling, does not always adhere to the same rigid time constraints as physical exercises. The nature of mental exercises can vary greatly, and they might not fit neatly into an hour-long block. For instance, a meditation session might be more effective if it lasts 45 minutes one day and 75 minutes the next, depending on the player's state of mind. Thus, allowing for continuous values in mental training provides the flexibility needed to tailor this crucial aspect of training to the individual needs and responses of the player. This granularity allows for more precise adjustments, ensuring that mental conditioning is as optimized as possible for on-court performance. Therefore, we imposed the following constraints:

(x_S), (x_G), (x_P) are integers, and (x_M) is continuous.

The MILP model for this study was implemented in Julia, a high-level, high-performance dynamic programming language for technical computing, using the JuMP package. JuMP is a domain-specific modeling language for mathematical optimization embedded in Julia. JuMP was utilized for its competence in solving the complex requirements of this study's training regimen optimization.

Results

The optimal training schedule is:

1. (x_S) : Hours dedicated to Serving Practice (integer)= 2 hours (per week)
2. (x_G): Hours dedicated to Ground Strokes Practice (integer)= 6 hours (per week)
3. (x_P) : Hours dedicated to Physical Conditioning (integer)= 4 hours (per week)
4. (x_M) : Hours dedicated to Mental Training (continuous)= 1 hours (per week)

Maximized Z: 54.5

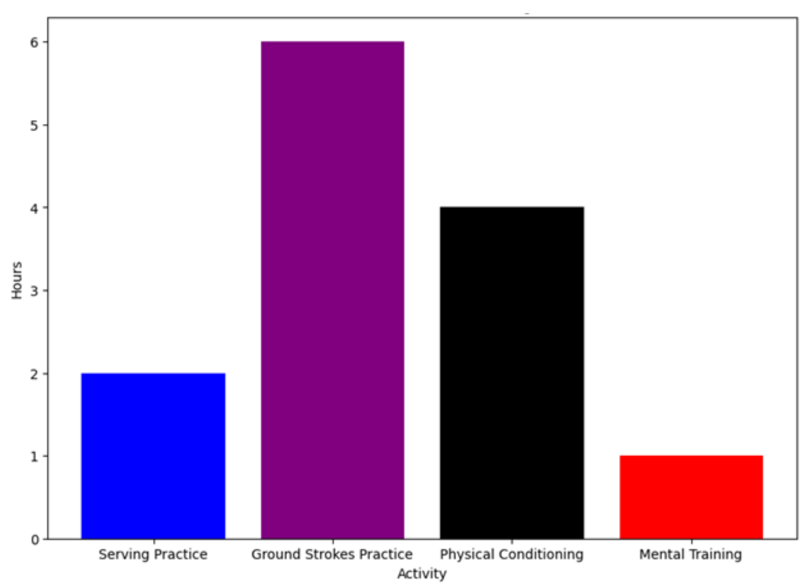

Figure 4. Optimal solutions for 4 activities

Table 1. Optimization Performance

Metric	*Values*
Matrix Range	[1,1]
Objective Range	[3,5]
Bounds Range	[0,0]
RHS Range	[1,20]
Objective Function Value	60
Confidence (Tolerance)	1.00e-04[1]
Best Bound	60
Gap	<01%[2]

[1]Indicates high accuracy
[2]Solution is optimal

The table 1 shows that the matrix range is uniform at a value of 1, suggesting that the constraints have consistent magnitudes and might not introduce scaling issues. The objective range spans from 3 to 5, indicating that the coefficients in the objective function are relatively close in value, which could potentially facilitate optimization. The bounds range is reported to be 0, implying that no specific upper or lower bounds have been set for the variables, defaulting them to standard ranges, often from 0 to positive infinity for both continuous and integer variables. Lastly, the right-hand side (RHS) range varies between 1 and 20, which denotes the spectrum of constraint values that the model must satisfy.

The model provided an optimal solution with a value (Z) of 54.5. This solution was determined with a confidence (or tolerance) of 1.00e-04, implying a very high accuracy. The objective and the best bound both matched at 5.450000000000e+01, leading to a very minimal gap value, which means the discovered solution is optimal. A total of 2.0 hours were apportioned to 'Serving Practice', 6.0 hours were dedicated to 'Ground Strokes Practice', Physical Conditioning consumed 4.0 hours, and finally, 'Mental Training' was allocated 1.0 hour. Given this allocation, the maximized value of the objective function, Z, was achieved at 54.5.

Conclusion

The study explores the utility of Mixed-Integer Linear Programming (MILP) in the optimization of tennis training regimens for prospective athletes in South Korea, more specifically the broader Ilsanseo-gu region in Goyang-Si, Gyeonggi-do. Considering the rising popularity of tennis in the region, it is paramount to investigate efficient training mechanisms that can give athletes a competitive edge. To build the optimization model, 48 prospective athletes in the range of semi-professional to ITF juniors were surveyed to establish the upper and lower bounds for four primary training activities, namely Serving Practice, Ground Strokes Practice, Physical Conditioning, and Mental Training. These bounds serve as constraints in the MILP model, ensuring that the suggested regimen aligns with realistic limitations and athlete capabilities. We also used multiple regression analysis to determine the relative importance of each of the four training activities. By formulating an objective function with these weights, we aimed to quantify the relationship between training hours spent on each activity and on-court performance. The MILP model then uses this objective function to allocate optimal weekly training hours within the established constraints. This allocation aims to maximize the value of the objective function, which in turn aims to maximize the athletes' on-court performance.

One notable limitation of the study is the relatively small sample size of 48 prospective athletes from South Korea. Although the sample size was sufficient for initial investigations, it may not fully capture the diversity and nuances of tennis athletes at different skill levels or from different cultural backgrounds. The study also employs multiple regression analysis which inherently assumes that the relationship between these activities and performance is linear. This assumption could be problematic as the actual impact of each training component on performance could be non-linear. For example, excessive emphasis on Physical Conditioning might result in fatigue that negatively affects Serving Practice, a detail not captured by the linear model of our study.

Limitations

In terms of accuracy, the methodology used in this study is, of course, inherently precise. This is because as all variables and coefficients within our model are integers, the resulting objective values are also integers, thereby eliminating the risk of small numerical errors that can occur with non-integer optimization.

The use of MILP could be perceived as overly complex given the current integer variables allow for only 36 possible combinations (2 x 3 x 3 x 2), a quantity manageable enough to evaluate through complete enumeration. However, such a view does not consider the full potential and scalability of MILP. While the current model is simple, when the model is expanded, integrating additional constraints such as the availability of tennis equipment and scheduling constraints for players would complicate the problem significantly—transitioning it into a complex scheduling and resource allocation challenge. These are scenarios where the brute force approach would be ineffective, yet our MILP approach maintains its efficacy.

Hence, the initial application of MILP in this study serves as a strong foundation for more complex future research to build off on. MILP's versatility will allow for more complex problems with new constraints and conditions.

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