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A Historical Review of Research on the Density-Size Correlation of the Observable Universe

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ABSTRACT

This literature review explores the correlation between the density and size of celestial objects - a key subject in understanding their formation, evolution, and the potential existence of higher-order clusters. It draws attention to the two seminal papers - E. F. Carpenter's density-size correlation study and Gerard De Vaucouleurs's study on matter density in groups and clusters. While acknowledging the limitations and outdated nature of Carpenter's body of work, it underscores the need for a more contemporary, comprehensive examination across various scales from neutron stars to galaxy clusters. The review highlights how such a study can clarify questions surrounding the size and shape of the universe, address Olbers' Paradox, and contribute to theories that describe the universe's origins and dynamics.

Introduction

To understand the density-size correlation, one must first understand the cosmic scale. Neutron stars, the smallest objects analyzed in related research, are around the size of a city, while galaxy superclusters, the largest analyzed objects, are clusters of clusters of galaxies, which themselves contain hundreds of billions of stars. The larger the cosmic object, the lower its density, which is the density-size correlation that is the topic of this study. There are currently only two research papers that focus on the density-size correlation, and they were both written before the Moon landing in 1969 (1,2). This study is an examination of these two research papers, theoretically determining what part(s) of their research are still valid today and what part(s) are outdated. From this information, it can be determined what further research is needed to fully understand the density-size correlation.

The concept of a homogeneous universe has existed long throughout the study of cosmology. Such an idea is essential for postulates such as the Cosmological and Perfect Cosmological Principles to be valid. However, there has also been a debate raging in the astronomical community for as long as the idea of such a structure for the universe has been entertained: the debate between the idea of a homogeneous universe and the idea of a hierarchical universe. Where a homogeneous universe considers the cosmos to be self-similar on a large enough scale, a hierarchical universe finds it displaying properties similar to that of a fractal. Similar, repeating structures between multiple reference frames at multiple scales would be found in a hierarchical universe. In a universe following the Cosmological Principle, average density of different locations at a big enough scale would be equal, and in a universe following the hierarchical model, there would be noticeable fluctuations in average density at the same considered scale. This roughly proves that a hierarchical universe is not homogenous. This has caused many disagreements as to which model fits the universe. On the one hand, at large enough scales, the universe appears to be homogenous. However, on smaller scales, the universe appears hierarchical. At this point, observation alone is no longer a viable means of determining the correct model, pointing to more theoretical ways of solving this problem. One such way this can be achieved is by looking at the size-density correlation across various scales for the universe. If a clearly linear correlation can be found, a hierarchical universe will prove to be true, therefore the universe would display an intrinsic repeating property such as density on varying scales.

Historical Overview and Common Findings

E.F. Carpenter's pioneering study on the "Characteristics of Associated Galaxies, I. “A Density Restriction in the Metagalaxy" was the first to lay the groundwork for the evaluation of density-size correlations of the universe. Carpenter introduced his methodology of calculating the density of galaxy clusters, defined by the count of thousands of nebulae per cubic megaparsec, using data fetched from stellar databases of Mount Wilson and Harvard.

As a core part of his research, he instigated the plot of a density-size graph derived from observed galaxy clusters and implemented linear regression analytics. Surprisingly, the data demonstrated more density in galaxy clusters with smaller radii and less density with larger radii, showing a negative correlation between the two variables.

From these results, Carpenter deducted a fascinating hypothesis: all galaxy clusters in the universe are interconnected. He presented galaxy clusters as the maxima and minima of a "nonuniform space distribution," limited by a density restriction (1). This groundbreaking concept still holds significant influence in contemporary astronomical papers exploring the universe's fractal distribution, signifying Carpenter's enduring impact on our understanding of celestial structure and interaction.

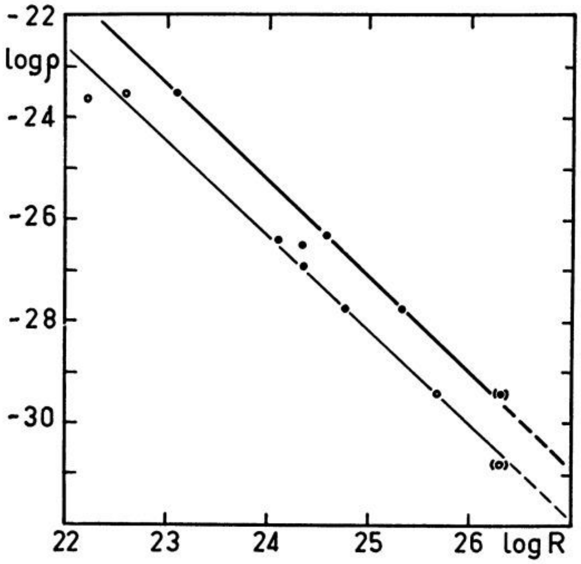

Figure 1. E. F. Carpenter’s Density-Radius Correlation (1).

Expanding upon Carpenter's work, Gerard De Vaucouleurs revisited the subject in the 1960s by introducing a new paper, “The Apparent Density of Matter in Groups and Clusters of Galaxies” (2). De Vaucouleurs explicitly outlined his process for ascertaining the relationship between maximum apparent mass and the radius of clusters.

The study was meticulous in details, citing all data sources and elaborating on how mass and density data was procured. Notably, a clear regression of his data was showcased, presenting a neat downward trend that further supports Carpenter's findings on the correlation between celestial object density and size.

By adding more groups, and correcting distances and their scale, De Vaucouleurs reworked Carpenter's study and defined an exact mathematical formula for the correlation. The formula derived was:

$$\text{Eq. 1. } Log\ N = 2.685 + 1.5\ logR = 2.375 + 0.logV$$

where N is the maximum number of populations, R is the Radius in megaparsecs (Mpc), and V is a cubic megaparsec.

The equation suggests a relationship between the number of galaxies a cluster can host and the cluster's physical size, implying a "density restriction" where there is a limit to how many galaxies can exist within a given volume or size, a concept pivotal to the understanding of the structure and evolution of celestial objects. This formula aids in understanding that larger clusters tend to host more galaxies, while smaller ones accommodate fewer galaxies.

De Vaucoulerurs continued past this, and eventually conducted another study, titled "The Case for a Hierarchical Cosmology" (3). In this study, De Vaucouleurs took data from multiple documented sources for all scales of celestial objects, not just from galaxy clusters and superclusters. What he found was an extended negative linear radius-density relationship that he saw could be split into two. He used this new relationship to argue for a hierarchical universe to become the new cosmological norm. For stellar bodies he defined the relation as:

$$\text{Eq. 2. } log\ p = -2.7\ (log\ R - 11.0)$$

where p is the density in grams per cubic centimeter and R is the radius in centimeters.
For larger cosmological objects he defines the relation as:

$$\text{Eq. 3. } log\ p = -21.7 - 1.7\ (log\ R - 21.7)$$

where, again, p is the density in grams per cubic centimeter and R is the radius in centimeters.

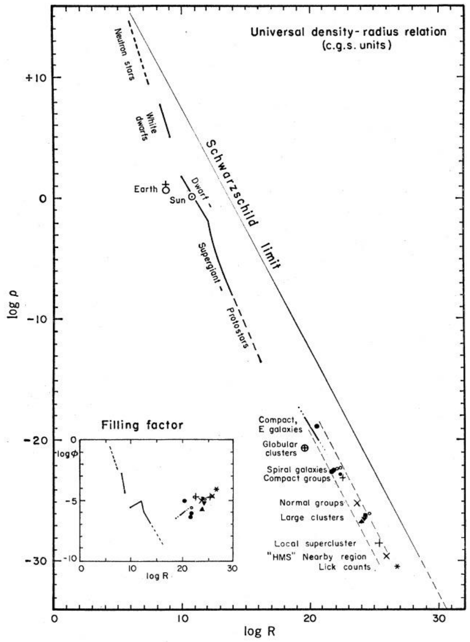

Figure 2. Gerard De Vaucouleurs' Universal Density-Radius Correlation (3).

These relations seemed to prove that the universe as a whole should be viewed hierarchically instead of homogeneously, as it was classified by patterns that repeated themselves on multiple scales.

Need for Future Research

Both Carpenter's and De Vaucouleurs' papers are outdated. Their papers were written in 1938 and 1960, respectively, and they are still cited today. In addition, Carpenter indexes his data with Shapley numbers, a classification system that has since been improved upon. On top of this, Carpenter's tools did not allow him to take super precise measurements (2). This does not mean his conclusions were wrong; instead, it only questions the exact precision of his data and calculations. Carpenter's paper was also very limited in scope, looking at 42 data points from only clusters of galaxies (1). In his two papers on the subject, De Vaucouleurs used relatively newer data from a wider range of categories (2,3), but he collected very few data points. The inclusion of more data entries would allow for the determination of the true extent of this correlation, as well as many theoretical conclusions.

Impact

The significance of De Vaucouleurs' and Carpenter's work lies in its profound impact on understanding the universe's structure, the distribution of matter, and the formation of galaxies. The density-size correlation presents an intriguing picture of universal structure, ascertaining a non-random, interconnected spread of galaxies. This work challenges earlier notions and pushes scholars and researchers to redesign theoretical models to accommodate the correlation observed.

Furthermore, these studies have significantly contributed to developing advanced statistical tools to assess and interpret the density-size correlation appropriately. This represents a significant leap from traditional methodologies used in astrophysical studies, broadening the scope of our understanding of celestial objects across multiple scales.

Lastly, the correlation bolsters the case for a hierarchically structured universe, underpinning the essence of theories of large-scale structure formations. Investigations leveraging this correlation have furthered our understanding of celestial dynamics and have led to a more coherent view of our place in the universe.

The exploration of the density-size correlation can provide valuable support for the de Sitter universe model. By studying the correlation between density and size across various scales, we can assess whether the observed patterns align with the expectations of a de Sitter universe. If the density-size correlation reveals a consistent trend of decreasing density with increasing size, it will align with the predictions of a universe dominated by dark energy. This alignment strengthens the case for dark energy's presence and significance in shaping the cosmos' expansion and structure.

Furthermore, understanding the density-size relationship can help us address the long-standing enigma known as Olbers' paradox, explaining why the night sky is not uniformly bright despite the vast number of stars in the universe. A density of negligible magnitude would prove this hypothesis. If the density-size model is used for the universe, it could contribute to the paradox by introducing the concept of limited density despite the vastness of the universe.

One of the plausible impacts of a hierarchical universe is how it impacts the curvature of space. According to the De Sitter model, once density gets negligible, space will essentially flatten. By leveraging findings from density-size correlation studies, one could theoretically predict when the line reaches a log of such a large negative number. This would be an invaluable tool for scientists to potentially pinpoint when a universe transitions from being curved to flat, or vice versa if a universe changes in the opposite direction.

Moreover, the evidence of a homogeneous universe could be demonstrated through the cosmic microwave background (CMB), the radiant heat left over from the Big Bang. Analysis of CMB radiation indicates that it is nearly identical in all directions, pointing to a homogeneous and isotropic universe on large scales. While this does not discredit the hierarchical structure at smaller scales, it suggests a reconciliation with the De Sitter model at cosmic scales.

Thus, the research on the density-size correlation, although dated, forms an important building block in our pursuit to comprehend the intricate patterns and behaviors of the celestial objects that populate the expanse of the universe. As we continue to push the boundaries of our knowledge, it is crucial to draw from the historical lessons drawn from past research and persist in our quest to decipher the secrets of the observable universe.

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