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Effects of Black Holes on the Space-Time Continuum

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ABSTRACT

Black holes are one of nature's greatest mysteries. For years, these celestial bodies have evaded scientists due to their intense gravity and extreme density. A specific area of black hole research that is overlooked is how these cosmic bodies affect the space-time continuum. Developing a complete understanding of space-time in and around black holes is essential for future research relating to black holes and combining the two great theories of this generation: general relativity and quantum mechanics. This paper aims to analyze how the space-time continuum is distorted by black holes and explain the applications that space-time around black holes has to modern physics. It first develops an understanding of space-time linkage and curvature through Einstein's theories of relativity and presents the geometry of space-time as predicted by Minkowski's equation. When inside a black hole, space and time essentially trade places so that the flow of time causes matter to be drawn within a black hole. As one explores further toward the centre of a black hole, curvature increases until it is predicted by general relativity to reach a gravitational singularity where density becomes infinite, and the laws of space-time break down. In the paper, the discrepancy between general relativity and quantum mechanics is explored through Hawking Radiation and the theoretical singularity at a black hole's centre. A combination of written explanations, math models, and diagrams are used to communicate how the space-time continuum is affected by black holes.

Introduction

Ever since Albert Einstein developed his theory of general relativity, black holes have been the subject of much speculation and research as physicists and the general public attempt to understand these mysterious bodies' nature. Black holes are an area of intense interest because they challenge Newtonian physics and, therefore, the conventional way of thought, altering the way society perceives space, time, and matter. Black holes are astrophysically defined as regions of space-time with intense curvature and mathematically defined as precise solutions to Einstein's field equations (Bambi, 2020). While much research has been done on the formation and properties of black holes, there is less information about how black holes affect the space-time continuum.

Space and time are relative, which combine to create a four-dimensional space-time, with three vectors for space and one for time. General relativity explains that gravity is a curvature of space-time. Regions of space-time with extreme gravity, such as black holes, also exhibit extreme curvature. There is a predicted breakdown of space-time laws at the centre of a black hole, which is explained in section 5; however, looking at black holes from a quantum mechanical perspective may solve this singularity (Borissova & Eichhorn, 2021).

Quantum mechanics describes small-scale matter, whereas general relativity explains massive objects (Peres, 2004). Discrepancies between quantum theory and general relativity are especially prevalent regarding black holes, due in part to the contrasting nature in which black holes exist. One alteration in quantum theory to encompass black holes may have significant implications for general relativity. This is because ignoring one and focusing on the other can cause extreme changes in the laws of physics. Therefore, understanding space-time around and within black holes, particularly the theoretical singularity, is a significant step toward merging the two theories. Combining the two theories has proven extremely difficult, with 16 major approaches to unifying quantum theory and general relativity proposed (Dewitt & Giampiero, 2008).

This paper will review space-time and how black holes disrupt it. The paper first introduces in Section 2 the concept of space-time and theories of space-time linkage. The paper then provides background information regarding black hole formation in Section 3 and the two theories often used to explain black holes: quantum theory and general relativity. It then goes on, in Sections 4 and 5, to explain how space-time around black holes is affected, focusing on the Kerr metric and the extreme curvature within the black hole until it reaches the centre where a singularity is predicted to lie. In Section 6, the paper concludes with the future applications in research of understanding space-time relating to black holes and possible routes for this research. The aim of this paper is to provide insight into how space-time is affected by black holes and provide a deeper understanding of space-time curvature in an effort to develop a unified theory of quantum gravity.

Space-Time Linkage and Warping

Newtonian physics describes the mathematical concepts of space and time as being absolute. Isaac Newton states that absolute, true, mathematical time flows without external relations. Newton also states that absolute, true, mathematical space is immovable without any external links, and relative space is the measure of the absolute spaces with reference to a system of bodies. Albert Einstein discovered that the Newtonian descriptions of space and time were not accurate and developed his special theory of relativity (Einstein, 1905), which was built on two postulates:

1. The Principle of Relativity: Physical laws are not affected by uniform translatory motions: they remain unchanged in reference frames with no acceleration.
2. The Principle of the Absoluteness of the Speed of Light: Any ray of light is stationary in a vacuum.

The relativity of space and time differs greatly from Newtonian absolute space and time only at speeds close to light. Some results of space and time relativity are length contraction and time dilation. Length contraction explains that a moving object is shortened in the direction of its motion, and time dilation demonstrates that a moving clock runs more slowly than a stationary clock (Wittman, 2018).

Special relativity assumes the existence of reference frames where Newton's first law holds: objects free of external forces will remain at rest or in uniform motion. These frames are called inertial reference frames. Lorentz transformations relate the coordinates in one reference frame to another. In these transformations, two reference frames are assumed where one person (measuring space and time using symbols x', y',z',t') is moving relative to another (measuring space and time using the symbols x, y, z, t) at a constant speed v along the x-axis.

$$t' = \gamma(t - vx), x' = \gamma(x - vt), y' - y, z' = z \quad (1)$$

where γ is defined as $\gamma = 1/\sqrt{1 - v^2}$

The Lorentz transformations formally express the concepts of relativity. They explain how space and time are not absolute and demonstrate how the speed of light is constant with respect to two objects moving relative to one another.

In 1908, Hermann Minkowski developed a four-dimensional geometric interpretation of Einstein's special relativity theory (Hehl, 2008). This four-dimensional "space-time" is absolute, meaning it is the same from all reference frames (Thorne, 1995). Minkowski's four dimensions consist of the three dimensions of Euclidean space, given by the variables x, y, and z; and one dimension for time, given by the variable t (Caroll, 1997). In Minkowski space-time, the four axes are defined as the vectors (x, y, z, ct), where c is the constant of light. The Minkowski metric can be used to find the space-time interval between nearby points using the equation:

$$ds^2 = -c^2dt^2 + dx^2 + dt^2 \qquad (2)$$

where s represents distance.

Through a math model of the space-time continuum, Minkowski demonstrated that space and time are interconnected and exist in a four-dimensional space-time fabric.

To explain how gravity is a curvature of space-time, Einstein used Minkowski's absolute space-time to create his general theory of relativity. Newton's law of universal gravitation views gravity as a force of attraction between two particles. While this description of gravity is sufficient for small masses and those travelling at low speeds, it is limited and does not accurately describe how gravity acts around massive objects such as black holes or at speeds close to light. At this point, one must look at gravity from a general relativity perspective. The theory of general relativity predicts that gravity is not a force but a manifestation of space-time's curvature (Einstein,1916). A massive object distorts the geometry of space-time around it, and this distortion controls the movement of physical objects (Einstein, 1916). General relativity consists of Einstein's field equations, which relate space-time geometry to the distribution of matter within it. Black holes are an exact solution to these equations and are explained quantitatively using metrics such as the Schwarzschild and Kerr solutions, which are described in Sections 3 and 4, respectively. Understanding the structure of space-time and the curvature manifested through gravity is essential for the study of black holes.

Black Holes

Black holes are extreme curvatures of space-time that significantly affect modern understanding of physical laws. Black holes were first described using the Schwarzschild metric only a few months after the publication of Einstein's Field Equations (Visser, 2007). The Schwarzschild metric describes the space-time geometry surrounding idealized, non-rotating, spherically symmetrical objects (Schwarzschild, 1916). One representation of the Schwarzschild metric is:

$$ds^2 = -\left(1 - \frac{2m}{r}\right)dt^2 + \left(\frac{dr^2}{1 - \frac{2m}{r}}\right) + r^2d\theta^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2) \qquad (3)$$

where r is the radius, m is the mass, θ and φ are angular coordinates in space-time.

The Schwarzschild solution predicts the formation of a physical parameter at $r = 2m$ (Heinicke & Hehl, 2015). This means that if an object collapses to a certain radius, there is a breakdown of Schwarzschild geometry. This parameter is called the Schwarzschild radius and is given using the equation:

$$R_s = \frac{2GM}{c^2} \qquad (4)$$

where G is the gravitational constant and c is the speed of light.

This parameter creates the event horizon of any non-rotating black hole. The event horizon was described by David Finkelstein in 1958 as a semi-permeable membrane, a membrane that can only be crossed in one direction (Finkelstein, 1958). This horizon is a defining property of black holes and is essentially the black hole's surface. The event horizon is a theoretical boundary where the escape velocity of a particle exceeds the speed of light.

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Three types of black holes have been identified: supermassive black holes, intermediate-mass black holes, and stellar-mass black holes. Supermassive black holes are found at the centre of galaxies and have masses from millions to billions of solar masses. These massive black holes have origins that may not come from the collapse of a massive star. The formation and evolution of supermassive black holes are not yet understood (Volonteri, 2012). Intermediate mass black holes are the rarest form of black holes, with only recent evidence pointing towards their existence, and have a mass of between 100-1^6 solar masses (Greene et al., 2020). Stellar-mass black holes are the most common types of black holes and will be the primary focus of this section. These black holes are formed through gravitational collapse: the contraction of an astronomical object due to the influence of its gravity. One must first understand potential options for the end of a star's cycle to understand the formation of stellar mass black holes.

According to the Chandrasekhar limit, a star with less than 1.4 solar masses can balance the object's gravity using the degeneracy pressure of electron gas inside the star: this star becomes a white dwarf (Chandrasekhar, 1934). A star with over 1.4 solar masses does not form a white dwarf since the electron degeneracy pressure can no longer withstand gravity. The star continues to collapse until the central region of the star crushes together every proton and electron into a neutron, creating some of the densest objects in the observable universe: neutron stars (Lattimer & Prakesh, 2004). The degeneracy pressure of neutrons in the neutron star balances gravity, preventing further collapse (Baade & Zwicky, 1934). If a neutron star is too massive, about 3-10 solar masses, then it can no longer resist gravity by internal pressure, and the object must implode to form a stellar-mass black hole (Oppenheimer & Snyder, 1939). Once a star collapses down to the Schwarzschild radius, gravitational collapse becomes inevitable and irreversible.

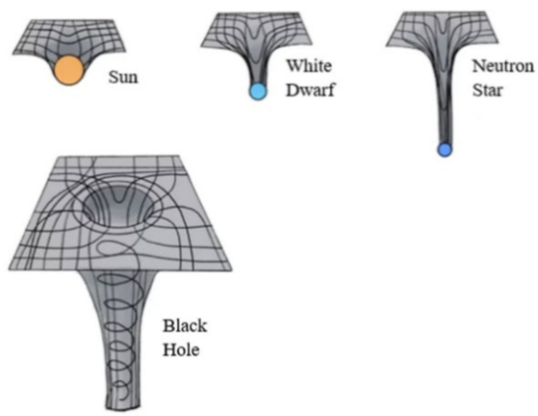

Figure 1. Space-time distortion by different massive objects. As an object's mass increases, distortion becomes more evident. Adapted from NASA.

According to the No-Hair Theorem, Black holes have only three measurable physical properties: mass, angular momentum, and electric charge. In classical general relativity, the solutions of Einstein's field equations only explain the curvature of space-time and do not explain the quantum effects of black holes. Our present description of black holes based on general relativity indicates a breakdown of quantum mechanics (Giddings, 2019). Stephen Hawking discovered one significant example outlining the breakdown of quantum laws in relation to black holes in 1974. Hawking demonstrated that black holes emit radiation by applying the laws of quantum mechanics to black holes. Hawking's calculations showed that the radiation left behind would be in a thermal quantum state and would only give information regarding the mass, angular momentum, and electric charge of the black hole, no matter the initial state of matter. This would mean the black hole eventually evaporates, with the initial properties of the black hole permanently lost (Hawking, 1975). A fundamental aspect of quantum mechanics is that information can never be lost, so black holes emitting radiation is a significant issue in quantum theory. To resolve this crucial problem in physics, called the information paradox, the laws of general relativity must be merged with the laws of quantum mechanics to create a law of quantum gravity.

Space-Time Outside the Event Horizon

To describe the geometry of space-time around a black hole, the Schwarzschild and Kerr (Kerr, 1963; Kerr, 2007) solutions will be discussed. This section will focus on the Kerr metric since the Schwarzschild solution does not account for angular momentum. The Schwarzschild and Kerr metrics mathematically describe the three significant aspects of space-time warpage around black holes: the bending of space, the distortion of time, and the spinning of space.

The Kerr metric describes space-time around an uncharged rotating body (Marsh, 2014). The Kerr metric is defined as:

$$ds^2 = -dt^2 + \sigma\left(\frac{dr^2}{\delta} + d\theta^2\right) + (r^2 + a^2)\sin^2\theta d\varphi^2 + \frac{2mr}{\sigma}(a\sin^s\theta d\varphi - dt)(a\sin^2\theta d\varphi - dt)^2 \quad (5)$$

where $\delta = r^2 - 2mr + a^2$ and $\sigma = r^2 + a^2\cos^2\theta$

The Kerr metric introduces a new parameter, a, representing angular momentum. If a is set to 0, then the Kerr metric can be simplified to a version of the Schwarzschild metric (Heiniche & Hehl, 2015). Since the Kerr metric includes angular momentum, it allows a better understanding of how space-time acts around black holes and the structure of a black hole. An important consequence of the Kerr metric is the creation of an ergosphere. The ergosphere is the area around a black hole's event horizon where the warping of space-time is strongest. Outside the ergosphere, some of the effects of black holes on space-time are still present; however, they have less impact. The area with negligible gravitational influence is called the quiet region. Black holes with a higher angular momentum and charge become more of a prolate spheroid, thereby increasing the ergosphere's area (Gao et al., 2023) This means increased angular momentum and charge increase space-time distortion outside the event horizon.

Near any object with a mass, gravity distorts space-time, causing space itself to bend. Because of the black hole's large mass, the bending of space is more extreme. Around a black hole, light is also deflected (Hsiao et al., 2020). The deflecting of light by objects with extreme gravitational influences is called gravitational lensing (Narayan & Bartlemann, 1997). The visible bending of light around black holes demonstrates the extreme curvature of space-time around the object. Black holes also exhibit gravitational time dilation. This phenomenon can be understood by imagining an object falling into a black hole: as the object approaches the black hole, the strong gravitational influences cause time to slow and eventually stop at the horizon. Objects in the presence of a massive rotating body exhibit frame-dragging; also called the Lense-Thirring effect. Frame dragging is an effect on space-time that is predicted by Einstein's theory of general relativity. It states that the field around a rotating object is non-static. The effects of frame dragging are less extreme near a weak field regime; however, around a black hole, frame dragging becomes so strong that all objects near the black hole rotate in the same direction as the black hole (Ricarte et al., 2022).

These extensive gravitational effects of black holes allow physicists to understand the properties of black holes and test the fundamentals of general relativity. By focusing scientific research on how gravity is affected by black holes and large-scale collision events, great progress can be made in the field of astrophysics, as it is described through multiple theories.

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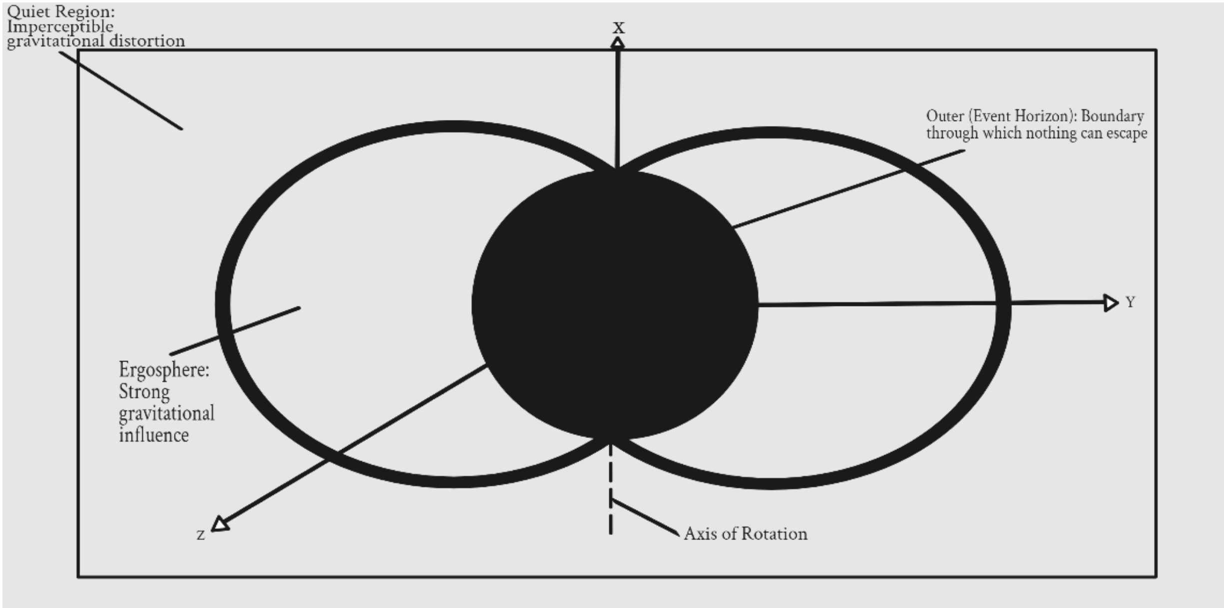

Figure 2. Two-dimensional depiction of space-time outside a black hole's event horizon as predicted by the Kerr metric.

Space-time Inside the Event Horizon

Because of a lack of observational evidence, space-time inside the horizon is much more complicated than space-time outside the horizon. The region between the event horizon and the black hole's centre has increased gravitational distortion, and the further towards the centre one ventures, the more gravity, and thus space-time, is distorted. Inside a black hole, time gradually becomes a space-like coordinate, whereas space becomes a time-like coordinate (Schutz, 2022; Hartle, 2021). This interesting consequence of general relativity occurs because the flow of time carries matter towards the centre of the black hole, causing in-falling matter to follow a time-like world line. A time-like worldline is a special curve in space-time where all space-time points can be rearranged to represent chronological events at the same place but at different times. Near the black hole's centre, it is conjectured that space-time should become extremely chaotic (Belinskij et al., 1970).

The greatest mystery regarding black holes is what lies at a black hole's centre. The Schwarzschild and Kerr metrics predict a gravitational singularity at a black hole's centre (Borissova & Eichhorn, 2021). Singularities in physics are areas where the classic laws of space and time break down, as do all the known laws of physics because they were formed on a classic space-time background (Hawking, 1976). The singularity of black holes predicted by the solutions to Einstein's field equations was not seriously considered until Roger Penrose published his singularity theorem in 1965. Penrose's singularity theory explained that once a surface where no light can escape is formed, a singularity is inevitable (Penrose, 1965).

The Penrose-Hawking singularity theorem proved purely through general relativity that a singularity is expected at the centre of every black hole (Hawking, 1970). The null version of the theorem generalizes previous results of the black hole singularity. It implies that space-time singularities are expected if there is a trapped surface where all matter follows the null energy condition, meaning it has a positive energy density. The theory can be simplified to state that a space-time singularity is formed in any trapped surface from which light cannot escape.

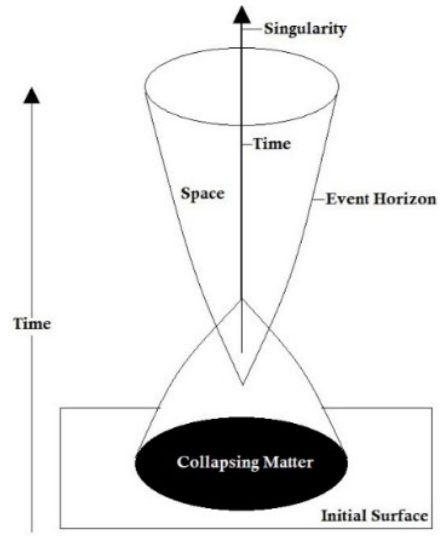

Figure 3. Space-time diagram depicting the gravitational collapse of a spherically symmetrical star to form a black hole. Adapted from Roger Penrose's diagram.

While the creation of a singularity is considered inevitable if one views black holes from a relativistic standpoint, it is suggested that the creation of a singularity in general relativity could be changed if the quantum effects are considered (Gambini & Pullin, 2008). A black hole's centre cannot be explained solely through general relativity or quantum gravity, and to understand how space-time acts inside a black hole, specifically what lies at the centre, a unified theory of quantum gravity must be created.

Conclusion

Space-time is a complex, interconnected continuum that is curved by massive objects such as black holes. Black holes are some of nature's greatest mysteries and will be a rich area of scientific research for years. These celestial bodies are important areas of research that have provided lots of insight regarding how space works and the extremities of space-time. Increasing an understanding of how black holes affect their surroundings is important for the field of theoretical physics as it can provide continued insight regarding gravitational effects at large and quantum scales.

Further research into black holes and how they affect space-time is required to blend the two great modern theories: quantum mechanics and general relativity. Future research should focus on understanding how black holes affect space-time from a quantum viewpoint and whether the results are consistent with relativistic descriptions. Once we can mathematically and geometrically describe the structure of space-time concerning black holes, specifically explaining a black hole's centre, a unification of General Relativity, Quantum Mechanics, and Newtonian Physics can be achieved, creating a Grand Unified Theory.

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