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Applications of Probability in Real World Examples

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ABSTRACT

Probability is one of the most important subjects studied in mathematics because of how useful it is in the modern world. On a daily basis, most people have to deal with something that involves probability. For example, some of the simplest things such as flipping a coin has to deal with probability. In this research paper, we will look at some examples of probability with applications to the real world.

Introduction

In this research paper, we start with some basic probability problems and then use those skills to apply them to interesting real-world applications of probability, such as the Birthday Problem, March Madness Perfect Bracket Problem, Deck Shuffling Problem, and the Slot Machine Problem.

In the case of the Birthday Problem, we investigate and try to find out how many people need to be in a room so that there is 50% chance or greater that two people in that room share the same birthday? The answer is that in a random group of 23 people (or more) there is greater than a 50% chance that at least two people have the same birthday. So, for example in a classroom of 23 students (or more) it is **likely** that at least two students have the same birthday. Moreover, we investigate and try to find out how many people need to be in a room so that there is a 70% and 90% chance that two people in that room share the same birthday? In a room full of 30 people, there is a greater than 70% chance of two people sharing the same birthday. In a room full of 50 people, there is a greater than 90% percent chance of two people sharing the same birthday.

In the case of the March Madness Perfect Bracket Problem, we look at the chances of getting a perfect bracket in the NCAA Men's Basketball Tournament and winning the one-billion-dollar prize. To win the Perfect Bracket, you need to pick all sixty-three games correctly in a single elimination college basketball tournament.

In the slot machine problem, we make up an example about a Las Vegas Slot machine. Using probability and expected value we calculate if this slot machine is worth playing if the purpose is to make money.

In the case of the Deck Shuffling Problem, we look at if you shuffle a standard card deck of (fifty-two cards), what is the probability that that deck in the exact order you have shuffled it has never been shuffled the same way before in human history.

Basic Problems

Problem 1. You flip a fair coin four times? What is the probability that at least three of those flips are heads?

Solution:
We can break this problem down into two simpler problems. What is the probability that you get 3 heads OR the probability of you getting four heads. Probability of getting three heads: 16 possible outcomes from flipping a coin four times (2•2•2•2 or 2^4). Four of those sixteen outcomes involve getting three heads. (THHH, HTHH, HHTH, HHHT). So, there are four favorable outcomes out of a possible 16 so the probability that there are 3 heads is 4/16.

Probability of getting four heads: 16 possible outcomes. Only one involves getting four heads (HHH) out of a possible 16 outcomes so the probability of getting four heads is 1/16.

Favorable Outcomes: 4 + 1 = 5
Possible Outcomes: 16

Probability = (# Favorable Outcomes) / (# Possible Outcomes)

Probability of getting at least three heads: 4/16 + 1/16 = **5/16**

Problem 2. There are eight red marbles in a bag and three green ones. You decide to draw two marbles. What is the probability of drawing two green ones?

Solution:
There are eleven marbles in the bag and three of them are green. The first step of answering this question is to break it up into two questions. What is the probability of getting a green marble on your first draw and then if that draw is green what is the probability of getting a green marble on your second draw. Drawing a green one gives you a 3(number of green marbles)/ 11 (total # of marbles chance) of getting a green one on your first draw. Now if your first draw was green then there are only 2 green marbles in the bag out of a total 10. The probability of drawing a green one from there is 2/10. So, the final probability is equal to the probability that the first draw is green multiplied by the probability that the second draw is green. This results to 3/11·2/10, giving a probability of **6/110**.

Birthday Problem

Consider n people and let $p(n)$ be the probability that two people share a birthday. We will consider the following two problems (open ended questions).

Problem 1. How many people need to be in a room so that there is a 50% chance or greater that two people in that room share the same birthday (365-day cycle)?

Problem 2. How many people need to be in a room so that there is a 70% chance or greater that two people in that room share the same birthday (365-day cycle)?

Solution of problem 1 and problem 2:
The answer is much lower than most think initially. To find the probability let us start with two people in a room. Some people guess things like "one hundred" but the answer is lower than that. The probability of person 1 having the same birthday as person 2 is 1/365. Now let us add another person to that room. The probability of person 3 not having the same birthday as person 2 is (364/365) and not having the same birthday as person 1 is (363/365). So you have to multiply the two probabilities that each of them do not share a birthday, which is (364/365) ·(363/365). For n people you keep repeating the same process (1- (1/365)) ·(1- (2/365))·(1- (3/365))·(1- (4/365))·(1- (5 /365))· … · n times. After you reach 23 people that probability reaches around 48%, saying that there is a 48% chance that two people DO NOT share the same birthday meaning that 100-48 = 52% chance of two people sharing the same birthday. When there are only 30 people the probability of two people sharing the same birthday is 70.6%. With 57 people in a room there is a 97% percent chance that two people share the same birthday, while with 100 people in a room the probability of two people sharing the same birthday is 99.99997% basically guaranteed that that two people share the same birthday.

People in a room	Probability that two people in a room share a birthday
1	0%
10	11.7%
23	50.7%
30	70.6%
50	90.7%
100	> 99.999%

March Madness Perfect Bracket Problem

Background: Every year, sixty-four of the top college basketball teams in the NCAA are invited to a single-elimination tournament where the winner advances until there is just one game left with the winner being crowned champion. People fill out brackets picking who they think is going to win the game in hopes of picking all sixty-three games correctly. If they are able to achieve that task, they receive 1 billion dollars from Warren Buffet. What is the probability that you will get a perfect bracket?

If we assume that every team is even, and it is a 50-50-coin flip then you will have a 50% (or ½) chance of getting each game right so the probability of getting all 63 games right is (½)^63. Which is 1 in 9.2233720368548•10^18 basically making it virtually impossible that you or anyone else will ever get a perfect bracket. However, all basketball teams are not the same skill and the best computer models have been able to predict March Madness games at 70% accuracy. At a seventy percent accuracy the probability of getting a perfect bracket is (7/10) ^ 63 chance which is gives a 1.7425•10^-8 chance of getting a perfect bracket even with predicting games at a seventy percent accuracy. Overall, the chance of getting a perfect bracket is slim and with predicting teams that have more skill. The billion-dollar prize is unlikely to be won and according to the NCAA website if people keep submitted brackets at the same rate, they will expect a perfect bracket 243 years from now.

Slot Machine Problem

Suppose you see a slot machine that costs $1 to play. There are 16 symbols and three slots and if you match 2 out of 3 symbols you get $3 and if you get all 3 symbols matching you get $100 dollars. The symbols are chosen randomly. Should you play this machine?

The easiest way to solve this problem is to simulate 16^2 or 256 times with average luck to see if you would make a profit. Total Cost of playing 256 times: $256•1(fee for playing) -> **$256**

Probability that you get 3 out of 3 matching symbols 1 * (1/16) • (1/16) -> 1/256 with playing 256 times you should average 1 time getting 3 out of 3: 1•$100 -> **$100** from this case.

Probability that you get 2 out of 3 matching symbols 3• ((15/16) •(1/16)) -> 45/256 with playing 256 times you should average 45 times getting 2 out of 3: 45•$3 -> **$135** from this case.

Playing 256 times should earn you $235($100 + $135) on average. The cost of playing 256 times is $256 which is greater than the expected output of $235. You should not play this machine if playing to make a profit because on average you will lose money.

Deck Shuffling Problem

You decided to shuffle a deck of playing cards. What is the probability that your shuffle has never been shuffled before by anyone in human history?

To start we must count the unique number of shuffles. There are 52 cards, the first card in the deck could be 51 cards, the second card can be 50 cards, the third card can be a so on giving a probability of 52!

This gives us approximately 8•10^67 unique ways a deck can be shuffled. Now we have to estimate the average amount of decks that people shuffle in their lifetime. The number does not have to be exact so let us estimate that the average amount of shuffles that a person does in their lifetime is 500 (this number doesn't need to be exact). The total number of humans in human history is 117 billion and multiplying that by the average number of shuffles in a lifetime (500 shuffles) gives us a total amount of shuffle of 117 billion • 500 = 58.5 trillion. The probability that your shuffle is unique is approximately (8•10^67 - 58.5 trillion)/(8•10^67) which is greater than 99.99999% meaning that when you shuffle a deck it is almost guaranteed that that exact order of cards you shuffled has never been shuffled before in human history.

Note. This problem assumes every shuffle is completely random. Humans probably shuffle cards in a certain way which would make shuffling cards not completely random leading to a greater probability than what we determined.

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