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Analysis of Existing Physical Theories Explaining Aerodynamic Lift Production by an Airplane

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ABSTRACT

Since the first successful airplane, invented by The Wright brothers in the early 1900s, air travel has become one of the most common modes of transportation. While airplane production and use rapidly increase, explanations regarding the physics of flight remain elusive due to major shortcomings. With all the research available, this paper aims to explore how accurately current theories have explained lift production by an airplane's wing. The contents of this paper will touch on the subject of physics, more specifically fluid dynamics and aerodynamics, and even more specifically, the aerodynamic force known as lift. Theories analyzed in this paper include the "Equal Transit", "Skipping Stone", "Venturi", Momentum-Based, Bernoulli-Based, and Circulation theory. Overall, limitations leading to the inaccuracy of these explanations include not providing physical explanations for specific assumptions, misusing and misinterpreting principles and theorems, simplifying explanations thus not accounting for all necessary factors, proposing one-way causation relationships, confusing mathematical theories for qualitative physical theories, and conflicting experimental data. Because of this, an additional purpose of this paper is to provide a more accurate and comprehensive explanation for lift.

Introduction

The motion of an airplane depends on the relative direction and strength of four aerodynamic forces. The force which causes an airplane to be elevated off the ground is known as lift. In aerodynamics, the physical mechanism lifting airplanes in flight is fundamental to accurately and explain to ensure flight safety and efficiency. The concept of lift, specifically how planes generate this force, is still a widely debated and arguably unsolved topic within physics. There is extensive research available on this topic; however, due to conflicting theories, there is still skepticism on what should be believed as valid and accurate. While lift theories may contain certain correct assumptions, the general shortcoming is in inaccurately proposing others (Landell-Mills, 2022).

Additionally, many theories fail to provide a full and complete explanation of lift that accounts for all forces, factors, and conditions that would affect lift. Popular explanations such as the "Equal Transit" and Newtonian theory of lift have been taught and printed in textbooks for decades. However, research has been conducted to disprove and contradict the theories. Then how do airplanes actually produce lift? Explaining lift is confusing as it has both technical and non-technical aspects. While both explanations are complementary, they aim to explain lift in different methods.

On the one hand, technical explanations consist strictly of mathematical theories. This includes figuring out the appropriate formula for a solution, like predicting the lifting force. The main aim of technical theories is to accurately predict and produce data that would be useful for aeronautical engineers when designing aircraft. However, equations are not explanations. Simply formulating an equation would not explain the physical forces that produce lift, hence, invalidating the solution for the purpose of explaining lift generation by airplanes (Regis, 2020).

On the other hand, a non-technical level of analysis or explanation intends to provide a qualitative physical explanation of lift. This type of explanation attempts to explain the physical forces applied to produce lift to provide an intuitive understanding of the factors and forces that work in lifting an airplane off ground. Rather than numbers, formulas, and equations, a non-technical level approach is based on principles and concepts which are applied to explain the concept. Commonly, misconceptions arise in these qualitative physical theories (Regis, 2020).

Moreover, experts in the field of physics, specifically aerodynamics and aeronautics, are still not able to conclude how lift is produced by airplanes. Hence, this paper aims to explore how accurately existing theories have explained lift production by an airplane's wing. To do so, the following sections of this paper will provide a more detailed description of lift. This will be followed by an in-depth evaluation and analysis of current theories of lift based on the strengths, limitations, and accuracy of the theory in describing lift. The theories that will further be discussed include the "Equal Transit", "Skipping Stone", "Venturi", Momentum-Based, Bernoulli-Based, and Circulation theory. By the end, this paper will provide a more accurate qualitative physical theory explaining lift produced by airplanes. Furthermore, the more accurate theory suggested in this paper will more validly interpret how lift is generated. That is to contribute a more comprehensive explanation of lift, which is highly beneficial to the field of aerodynamics and aviation.

What Is Lift?

Four aerodynamic forces act upon an aircraft during flight: thrust, drag, weight, and lift. As forces are vector quantities, they have both direction and magnitude, which determine the overall movement of the object. Thrust is the force produced by the propeller or engine of an airplane that forces the object forwards in the direction of motion. Drag is the force acting in the opposite direction to thrust. Drag slows down an airplane's forward movement, typically caused by friction, retarding force, and air movement. The downward force directed to the center of Earth is weight, the force of gravity. An airplane cannot leave the ground unless a larger force counteracts the gravitational effect of weight. This counterbalance force is lift, which has an equal magnitude and opposite direction to weight (Newton, 2021). An airplane's motion depends on the four forces' relative direction and strength. When the forces are balanced, equal opposing forces, the aircraft moves at a constant velocity through the air. When forces are unbalanced, the aircraft accelerates in the direction of the largest force.

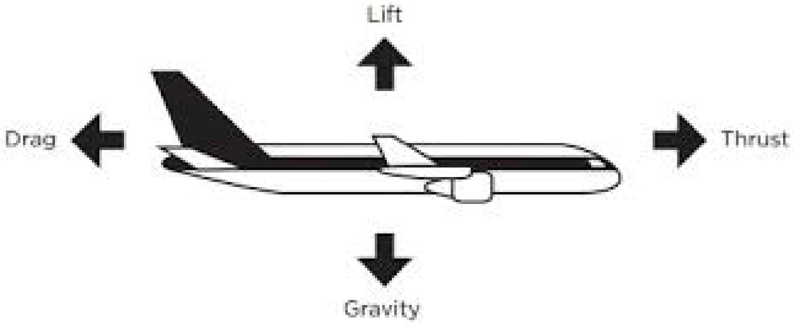

Figure 1. The four forces of flight. Figure taken from (*Four Forces of Flight*, n.d.).

Focusing more specifically on lift, the mechanical aerodynamic force moves perpendicular to the flight direction and is mainly generated by an airplane's wings. Wings are airfoils attached to the sides of an airplane's central body, also known as the fuselage. They are the main lifting surfaces that support an aircraft in flight. Looking from the top of a wing, the front of the wing is the leading edge, while the back is the trailing edge. A cross-section through the wing, perpendicular to the leading and trailing edges, will reveal an airfoil shape. A wing's airfoil has a specific shape commonly associated with generating lift. It is designed to provide the most favorable airfoil efficiency—lift to drag ratio—relative to the surrounding air. A theoretical straight line connecting the leading and trailing edges, called a chord line, splits the airfoil into an upper and lower surface. The centerline between the surfaces is known as the camber line, which describes how curved an airfoil is (Federal Aviation Administration, 2016).

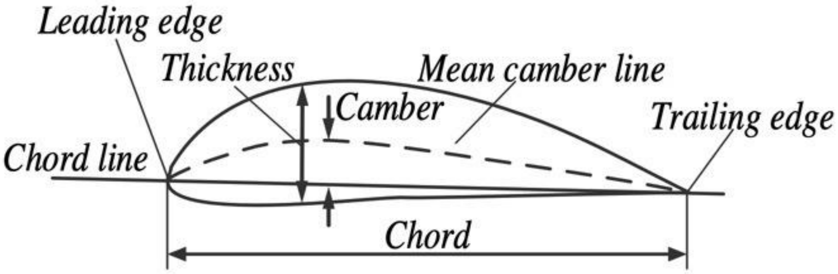

Figure 2. Airfoil of a wing. Figure taken from (Nguyen et al., 2015).

Current Theories of Lift

The understanding of how planes produce lift needed for flight has been extensively researched in physics. Currently, in the field of aerodynamics, there have been various attempts to theorize how airplane lift is generated. The numerous discoveries have contributed to a better understanding of lift produced by airplanes. With notable attempts to explain the concept, many theories are flawed and inaccurate due to misconceptions. This leads to the debate on which approach most accurately describes how lift is produced. Additionally, with advancing technology and research, conflicting qualitative physical assumptions have contradicted these stories. To understand where existing theories fall short in the explanation, this section will analyze several popular theories. These theories include the "Equal Transit", "Skipping Stone", "Venturi", Momentum-Based, Bernoulli-Based, and Circulation theory. Furthermore, an in-depth analysis of the individual theories will help identify common shortcomings which can be avoided when suggesting a more accurate physical theory.

"Equal Transit" Theory

One of the most popular theories of lift, taught and believed by many, is the "Equal Transit" theory. Also known as the "Longer Path" theory, it revolves around the movement of air molecules around a wing. First, it is essential to understand the airflow when an object moves through it. When undisturbed, air moves in smooth streams. However, when a wing moves through it, the oncoming airflow is disturbed, dividing the flow around a wing. The theory explains that an airfoil shape has a more convex upper surface than the lower. This results in the upper surface having a longer length than the bottom surface. The key argument is that air molecules on the upper surface travel a greater distance than on the lower surface. It assumes that two neighboring fluid particles which split at the leading edge should reach the trailing edge simultaneously. To do so, the molecules traveling above the airfoil require a greater average velocity than those traveling below the wing (Wild, 2021). Bernoulli's principle of differential pressure is then quoted to further explain lift. The principle states that an increase in the speed of a moving fluid occurs concurrently with a decrease in the pressure within the fluid. Thus, the airflow on top of the wing has less pressure due to the faster movement as opposed to airflow below the wing. Air pressure on the lower surface has more force than on the upper, pushing the wing upwards. Furthermore, due to the curvature of an airfoil, the pressure difference between higher pressure air below the wing and lower pressure air above the wing produces lift (Tennekes, 2009).

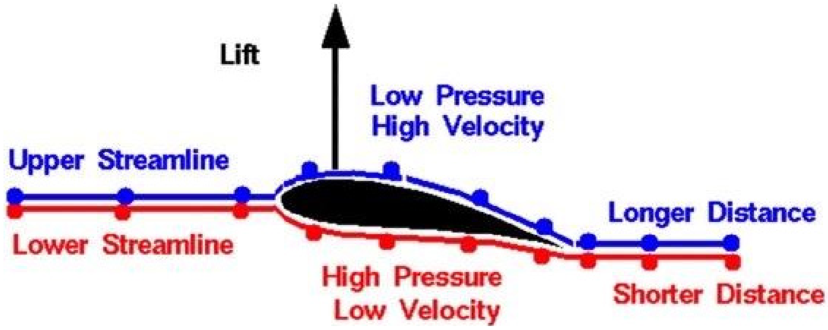

Figure 3. Theoretical illustration of the "Equal Transit" theory of lift. Figure taken from (*Incorrect Theory #1,* 2021).

While the theory does have some valid points, there are a multitude of limitations making it incorrect. First, the approach is based on a false assumption regarding the airflow around a wing's airfoil. Fluid particles that split at the leading edge of an airfoil do not have to transit to the trailing edge in equal time. Babinsky (2003) conducted wind tunnel experiments to verify the assumptions made in this theory. The experiment used smoke to visualize streamlines, a path traveled by a fluid particle as it moves with the flow, that would represent airflow around the wing in real life. Illustrated in Figure 4, when the flow around an airfoil was recorded, it was observed that the transit times around the wing were unequal. In reality, the flow, which started at the leading edge together, did not simultaneously meet at the trailing edge. Moreover, data shows that the airflow on the upper surface of a wing reaches the trailing edge before the time the flow below the wing reaches the same edge (Babinsky, 2003). The real-life experimental data contradicts the assumptions and claims made in the “Equal Transit” theory, hence, disproving it.

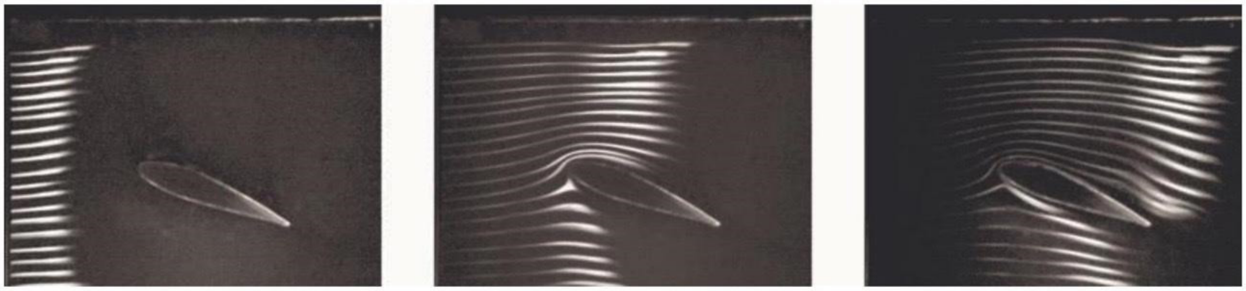

Figure 4. Wind tunnel data of smoke particles flowing along an airfoil. Figure taken from (Babinsky, 2003).

A second shortcoming to the theory is that Bernoulli's principle can only be applied in steady flowing, inviscid, and incompressible fluid. A fluid under steady flow means that the velocity does not change with time at a particular fixed point. Incompressible fluid means it can be considered that the density of a fluid is constant within a fluid parcel. Only being applied to non-viscous fluid flow means that the effect of viscosity is omitted (Hoffren, 2001). The theory applied Bernoulli's principle in its explanation to describe the velocity and static pressure of the flow around a wing. However, applying the principle would not be accurate in all circumstances. While the viscosity of air is small, it is not nonexistent. Therefore, ignoring it altogether would be inaccurate. Additionally, in fluid dynamics, lift cannot be produced without drag, and drag cannot occur in inviscid fluids.

While airflow can be under steady flow, incompressible, and inviscid, this is not true for all situations (Abbot and Doenhoff, 1949). Thus, it would be inaccurate to apply Bernoulli's principle to explain lift production because it does not account for fluids that are not perfect.

The theory also is flawed in the assumption that an airfoil needs to have a longer upper surface as opposed to the lower surface. Wings can generate lift with no difference in surface path lengths, such as symmetrical airfoils. Additionally, airfoils that only consist of a camber line and no wing thickness can be set at an ideal angle of attack to produce lift. This ideal angle allows airflow to meet at the leading edge and exit at the trailing edge on the upper and lower surfaces with the same transit length. Assuming the theory is true would mean that airfoils with different path lengths would generate drastically different lifting forces. That is, an airfoil that has a larger path length difference would produce more lift than an airfoil with a smaller path length difference. However, in real life, when airfoils have drastically varying path length differences in the upper and lower surfaces, the difference in lifting force produced tends to be very small. Therefore, to explain that the "Equal Transit" theory exclusively produces lift would be incorrect to account for the lift produced by airfoils (Craig, 1997).

Additionally, a real-life phenomenon that conflicts with the theory is that airplanes can fly inverted. To lift from the ground, the "Equal Transit" theory explains that air pressure on top of the wing must be less than that on the bottom. This occurs due to the convex upper surface of the airfoil, which during inverted flight, becomes the bottom surface. Thus, the greater pressure on top of the inverted wing, added with gravity, should result in more weight, downwards force. However, this is not true because airplanes can fly inverted (Newlands, 2016). Overall, while Bernoulli's principle is correct in explaining the relationship between air velocity and pressure, it is misinterpreted in the "Equal Transit" theory to explain lift, making the theory highly inaccurate.

“Skipping Stone” Theory

Another theory of lift that has attempted to explain the notion is called the "Skipping Stone" theory. As the name suggests, the physics behind this explanation is based on and somewhat similar to the way a flat stone or rock, thrown at an angle, skips across a body of water. The theory is based on the assumption that lift is a reaction force present due to an action caused by air molecules (McLean 2018). This phenomenon is known to be Newton's third law of motion, which states that to every action there is always an equal and opposite reaction. Newton theorized that all forces occur in pairs equal in magnitude but opposite in direction. While the law does not directly explain lift production, it is used to support this theory. Applying Newton's third law, lift is described as the reaction force to air molecules striking the bottom surface of a wing's airfoil when the object moves through the air. Similar to other theories, the wing is an obstruction to oncoming airflow. Thereby, the theory proposes that when airflow reaches the wing, molecules hitting the bottom of the wing are deflected downwards. The molecules hitting the lower surface of the airfoil transfer momentum into a reaction which is the lift force (Alexander, 2017). Lift, in this theory, is created by the wing's lower surface, while the upper surface contributes to little to no lift. This is because air flowing above the wing travels straight and undisturbed.

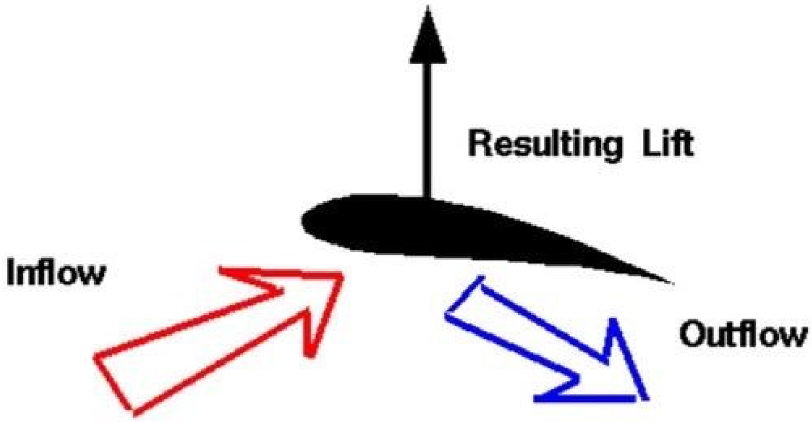

Figure 5. Theoretical illustration of the "Skipping Stone" theory of lift. Figure taken from (Incorrect *Theory #2,* 2021).

This theory, similarly to those touched upon in this section of this paper, has been disproven and is labeled as an incorrect theory. The first shortcoming of this theory is that it disregards the airflow above the wing and the molecules interacting with the airfoil's upper surface. It assumes that only the bottom surface produces lift, neglecting the upper surface, which significantly contributes to lift production (Blazevich, 2017). Looking back at wind tunnel data illustrated in Figure 4, the molecules around the wing bend closer to the airfoil. Air molecules above the wing are disturbed, thereby dismissing the upper surface of the airfoil would be inaccurate (Anderson & Eberhardt, 2010). Moreover, the theory assumes that air molecules above the wing are undisturbed and do not directly interact with the upper surface to generate lift, which is incorrect.

Additionally, the shape of the airfoil, specifically the upper surface, is not accounted for in the theory. Air molecules also interact with the wing's upper surface because it is not a vacuum. Through this assumption, it is expected that airfoils with different upper surfaces but an identical lower surface would generate the same lift. This is false and unrealistic because many airplanes install small plates on the upper surface called spoilers. These devices' purpose is to disrupt the airflow over the wing, changing lift to maneuver an aircraft (Abdelrahman, 1994). The theory does not address this feature which can affect lift. Therefore, assuming that airfoils with a different upper area and equal lower area produce the same lifting force is known to be invalid.

While this is a flawed theory, it is not entirely inaccurate. Under specific flight conditions, the approach is accurate in explaining lift. Fewer air molecules strike the airfoil's upper surface only when the wing travels at high speed and altitude. However, these flight regimes only occur during the early phases of a space shuttle re-entering the Earth's atmosphere. This theory would be a somewhat valid explanation for lift at hypersonic conditions requiring velocities above 16,000 km h^{-1} and altitudes above 80 km. However, most flight conditions fly around 800 km h^{-1}, 10 km above ground, making the theory inaccurate in explaining a plane's lift (Ding, 2021). The theory flaws in the assumption that most of the lift force is provided by air molecules being deflected against the wing's lower surface. Furthermore, the theory does not provide the most accurate explanation for the production of lift by an airplane's wing.

"Venturi" Theory

The next theory of lift is commonly known as the "Venturi" Theory due to its association with the Venturi tube and effect as an explanation for lift. Like most of the different theories, the wing acts as an obstacle to oncoming airflow. Because the flow cannot travel undisturbed through solid bodies, it must move aside, flowing around the obstructing body. Airflow avoids the wing by pinching a stream tube that behaves similarly to fluids narrowing within a Venturi tube. The theory assumes that the upper surface of a wing's airfoil is more convex than the lower surface. Due to this characteristic, the airfoil's upper surface is described as a "hump" that is a larger obstacle to the flow than the lower surface. The particular shape of the upper surface of an airfoil thus acts as observed in a Venturi tube (Escudier, 2017). A Venturi tube, depicted in Figure 6, is an instrument used to measure fluid flow in pipes with a constricted inner surface. The tube is composed of a cylindrical inlet the same size as its outlet. Between these entrances is a convergent throat which causes the fluid's velocity to increase while its pressure decreases (Boyes, 2010). This is known to be the Venturi effect and is an example of Bernoulli's principle of differential pressure. When the principle is applied to an airfoil, as a result of the conservation of mass in fluid flow, the upper surface is described to contract and accelerate airflow above the wing. The streamline traveling above the wing is pressed together more than those moving below the wing. Because of this, compressed air flowing above the cambered wing, like in a Venturi tube, results in air traveling at faster speeds which causes a decrease in pressure, creating lift.

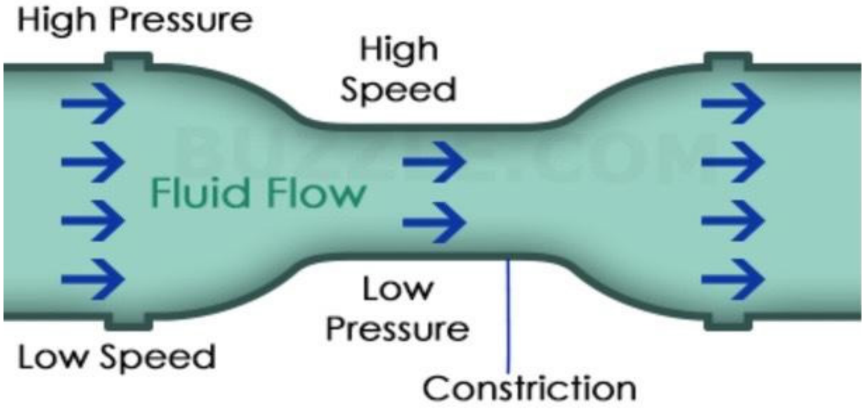

Figure 6. Venturi tube and effect. Figure taken from (*Explanation of the Venturi Effect*, n.d.).

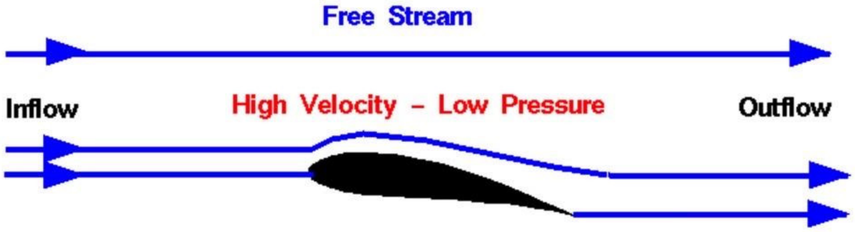

Figure 7. Theoretical illustration of the "Venturi" theory of lift. Figure taken from (*Incorrect Theory #3,* 2021).

While the principles applied in this theory, such as the Venturi effect and Bernoulli's principle, are correct, they are misinterpreted to explain lift by an airplane's wing. A first shortcoming of the theory is that it is based on the idea that the upper surface of an airfoil is shaped and acts like a Venturi tube. For the Venturi effect to be accurately applied, there must be an additional surface to create the other half of the tube. The theory only accounts for the upper surface of an airfoil, half of a tube. As the inlet, outlet, and constricted inner surface of a Venturi tube are not created, assuming that the airflow will behave like what is observed in a Venturi tube is incorrect as an airfoil does not have the characteristics of a Venturi tube (Liu, 2021). Furthermore, this explanation is false as an airfoil is not a Venturi tube.

A second limitation of the theory is that it fails to physically explain the claims made. The theory does not explain how stream tube pinching arises, how high-velocity flow above the airfoil occurs, and why the pinching is greater above the wing rather than below the wing. It is explained that, similar to the flow inside a Venturi tube, the airflow travels more pressed together than below the wing. Although an airfoil is not a Venturi tube, thus, the same principles cannot be applied to explain lift. Simply stating that airflow will sense an airfoil's upper surface would be a larger obstacle for oncoming air and that airflow above the wing will pinch together more for it to move at faster speeds does not state physically how this is possible (Anderson, 2008). This explanation only predicts that because of the convex upper surface of an airfoil, airflow will be more pinched together, causing the flow to travel at a faster velocity than on the bottom surface. However, the physical principle that contracts the airflow over the wing is still unknown. Moreover, explaining this assumption requires delving deeper into the dynamics of fluid movement (McLean, 2013).

Unlike the "Skipping Stone" theory of lift which only deals with the bottom surface of a wing, this theory neglects the airflow around the bottom surface of the airfoil. The theory states that airflow above the wing pulls the wing upwards, creating lift. In the theory, there is no mention of the effect of an airfoil's lower surface. If the theory were correct, a wing with an airfoil with any shape as its lower surface would produce the same lift force if its upper surface were the same. Not taking the lower surface into account flaws the theory as it does contribute to the lift produced by a wing. In most cases, the airfoil's bottom surface also obstructs the flow. Even if it is less than the top of the airfoil, it will still affect the airflow around the wing, which contributes to lift (McLean, 2013). A further extension to make this theory more accurate would be explaining that due to the larger obstacle on the top of the wing, high-velocity, low-pressure airflow travels above the wing. In contrast, low velocity, high-pressure air travels below the wing. Claiming that the pressure difference better the two surfaces produces lift would be a more accurate explanation for lift.

Another limitation of the theory is that it cannot explain lift produced by a flat wing, one with no thickness. The explanation argues that for lift to be generated, there must be constriction at the airfoil's leading edge. This constriction is described to accelerate airflow while its pressure decreases, pulling the wing up and creating lift. In a flat wing, no compression is present, meaning the Venturi tube is not formed. According to the theory, the constriction provided by the airfoil's upper surface would allow airflow to travel at high velocity and low pressure above the wing. Therefore, in theory, a wing with no thickness would not be able to lift off the ground. This is a limitation as, in reality, flat wings are able to produce lift (Xia and Mohseni, 2013). Furthermore, being based on the idea that an airfoil's upper surface behaves like a Venturi tube already sets the theory up for failure. Because the two objects do not have common characteristics, the principles of a Venturi tube cannot be applied to an airfoil. Furthermore, while the principles used in this theory were correct, it is misinterpreted, making the theory inaccurate in explaining lift.

Momentum-Based Theory

This following theory, called the Momentum-Based theory, involves Newton’s laws. Going back over the basics, Newton's first law states that an object will not change its motion unless an external force acts upon it. This suggests that a force must be acting to bend airflow or air initially at rest, accelerating into motion. An alternative form of Newton's second law used in this theory is that lift is proportional to the amount of air diverted down times the downward velocity of that air. As the two laws suggest, a change in the air's momentum is a direct cause of forces acting upon the wing (Anderson & Eberhardt, 1999). The theory argues that air is deflected downwards by the wing to be pushed up. This is because Newton's third law of motion states that every action has an equal and opposite reaction. As airflow splits around the wing due to it being an obstruction, the flow is described as leaving the airfoil's trailing edge at a downwards angle called downwash. This downward-traveling air is associated with producing lift by a wing (Anderson & Eberhardt, 2001). The action of an airfoil deflecting incoming airflow downwards creates a reaction force that pushes upward back on the wing. As a result, an additional force, lift, is imparted opposite to the downward-traveling air.

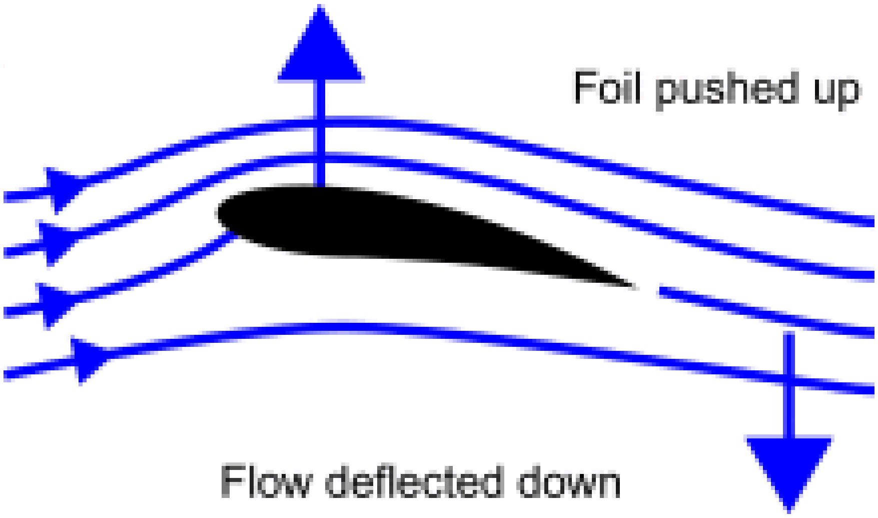

Figure 8. Theoretical illustration of the Momentum-Based theory of lift. Figure taken from (Breeze, 2016).

The Momentum-Based theory does not directly attribute a pressure field to generating lift like many other theories. This raises a question about the physical force or mechanism that deflects the airflow as a source of lift. On the lower surface of the airfoil, at a positive angle attack, the airflow is deflected downwards due to the surface being at an incline. On the upper surface, the Coanda effect is presented as an explanation of flow turning and how airflow can travel curved around the airfoil (Silva and Soares, 2010). The Coanda effect describes the tendency for a flow along a solid surface to attach and follow the surface. Viscous forces in the boundary layer make the flow turn toward the airfoil, and the difference in flow speed within the boundary layers leads to shear forces causing flow turning (Anderson & Eberhardt, 2001). The theory explains that the deflection of airflow and the flow over the upper surface contribute to the overall downward deflection. The action force of downwash then causes the reaction force of lift (Liu et al., 2015).

In hindsight, the Momentum-Based theory appears to be accurate due to its intuitive explanation of lift. Newton's laws provide a comprehensive description on a non-technical level, making it simpler to understand and accept. However, like many other theories, it falls short in providing a complete explanation of the assumptions made. According to Newton's laws, the theory claims that lift is produced by the deflection of the flow momentum flux. Causing the deflection at the trailing edge of the airfoil requires flow turning, which is described to occur like a Coanda effect (Saeed and Gratton, 2011). However, applying this effect to an airfoil would be inaccurate. The Coanda effect refers to the tendency for a jet flow to attach to a convex surface and follow the surface's outline. Airflow attaching to the upper surface appears like a Coanda effect; however, it is not. Although the boundary layer on the surface of an airfoil, although also a shear layer, is not the same as a powered jet (McLean, 2013). Thus, the Coanda effect cannot explain the flow above the wing.

Another flaw to the theory also regarding the application of the Coanda effect to explain flow turning and downwash on the upper surface of the airfoil. The effect implies that viscosity is a cause of flow turning about a curved surface. Combined with the flow turning in the direction of the slowest layer due to shear forces caused by a difference in flow speed within the boundary layers, the Coanda effect is used to explain the flow pattern above the wing (Barlow et al., 2009). This assumption is incorrect as there is no need for a relationship between the flow attaching to the curved upper surface of an airfoil and shear forces. Additionally, the theory provides two different explanations for the downwards deflection of airflow on the top and bottom surfaces of the airfoil. A consistent physical answer is not provided as two unrelated properties are proposed (Liu, 2021). Moreover, the application of the Coanda effect in this theory is incorrect in explaining the flow attaching to the upper surface to cause downwash.

Additionally, the theory is flawed as it cannot explain a lower pressure region observed above the wing. Over the years, cumulative distribution function (CDF) analysis has shown that a low-pressure air region appears on the top of the wing regardless of whether the airfoil has a cambered upper surface. As illustrated in Figure 9, in most wings during flight, there is always a reduced pressure area above the wing, which, in this theory, is disregarded. Because this low-pressure area is an unavoidable factor to lift, it needs to be considered (Regis, 2020). Furthermore, while this theory provides a more intuitive, universal, and comprehensive explanation of lift, it is still lacking in some areas making it inaccurate.

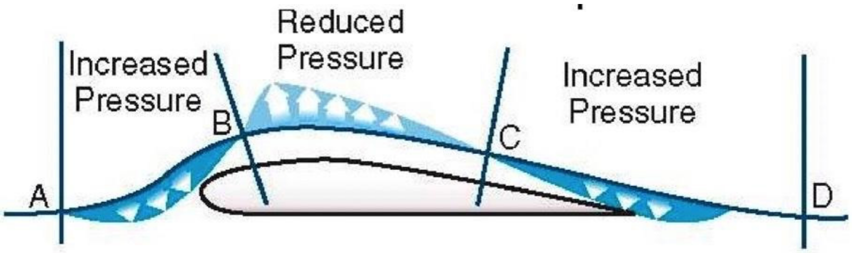

Figure 9. Pressure distribution above an airfoil. Figure taken from (Pawar and Sonara, 2017).

Bernoulli-Based Theory

As suggested in its name, the following theory explaining lift produced by an airplane's wing is based on Bernoulli's principle of differential pressure. The theory bases lift on the airflow around the wing, which follows Bernoulli's principle. Bernoulli's principle explains that within a horizontal flow of fluid, the acceleration of a fluid co-occurs with a drop in its static pressure. This also means that higher-pressure fluids move at slower velocities (Weltner, 1987). Moreover, the principle explains the relationship between a fluid's velocity and static pressure. Therefore, when velocity in fluid steam increases, the pressure decreases, and when the velocity decreases, the pressure increases (Thavamani, 2016). Therefore, the faster the flow, the lower the pressure.

Back to an airplane in flight, the theory assumes the flow around a wing is steady, non-viscous, and incompressible. The flow outside the outer flow and boundary layer act non-viscous. Therefore, Bernoulli's principle can be applied under steady conditions to the flow along and between streamlines (McLean, 2018). The fluid in this situation is air, composed of invisible gasses that have mass made up of molecules that exert air pressure. Because air molecules move in rapid motion and are not closely bound together, they can flow and move freely around obstructing objects like a wing. Thus, when a wing is flying through air, the flow is disrupted and divides around the body.

Depending on the wing's shape and angle of attack, the speed and pressure can change depending on where the molecules are with respect to the wing. The theory explains that the convex upper surface of the airfoil makes airflow speed up above the wing than on the bottom (Federal Aviation Administration, 2016). The flow is said to create an area of high velocity that forms above the upper surface of the airfoil. Then as a consequence of Bernoulli's principle, high velocity over the wing implies, or causes, low static pressure in the same region. Below the wing, lower velocity causes a high-pressure area to form. The high pressure below the wing exerts an upward force toward the low-pressure area above the wing as the high-pressure flow below the wing pushes the wing in that direction. This overall reaction produces lift. Furthermore, lift is produced by the pressure difference above and below the wing (Eastlake, 2002).

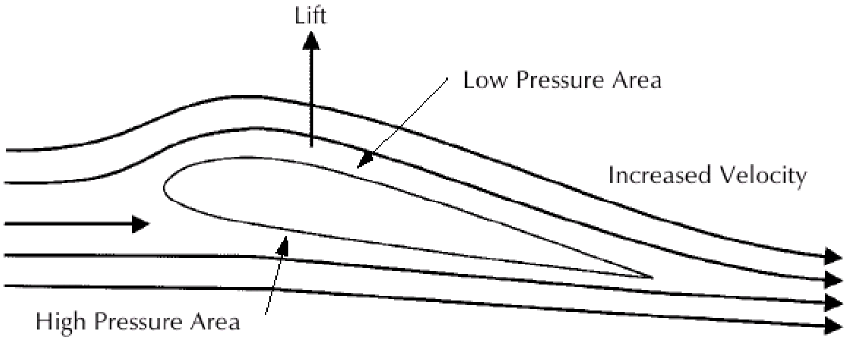

Figure 10. Theoretical Illustration of the Bernoulli-Based theory of lift. Figure taken from (Rahman and Omar, 2019).

As mentioned before, Bernoulli's principle is correct; however, it does not validly explain what causes the upper surface flow to travel faster, as explained in the theory. Simply stating that the air molecules traveling above the wing must travel faster than those on the bottom does not explicitly explain how the flow moves faster (Weltner, 1987). While inaccurate, the "Equal Transit" theory explains that airflow accelerates over the airfoil because the two neighboring fluid particles, which split at the leading edge, should reach the trailing edge simultaneously. However, the physical force accelerating the airflow to a higher velocity in the Bernoulli-Based theory is not identified. For the claim that air travels faster on top of the wing as opposed to the bottom to be accurate, the theory needs to identify the physical force that does so. Moreover, this theory is problematic in the sense that it does not provide a valid physical explanation of the assumptions made (Liu, 2021).

Additionally, the theory implies that the velocity of the fluid causes the corresponding pressure. Although it is not explicitly stated, it is implied that there is a one-way causation in which a fluid's velocity causes pressure which is a misconception. It is incorrect to assume that velocity needs to come first to cause and establish a pressure field as a result. This is because almost every cause and effect relationship, including that between flow velocity and static pressure, is reciprocal. This connects back to the prior limitation as pressure is seen strictly as a result of velocity and not as a potential cause. The theory fails to explain what causes high-velocity airflow on top of the wing. It attempts to explain this without accounting for the pressure as a cause due to it being implied as a result. While fluid velocity and static pressure are necessary for lift, the assumption that it is one-way causation in which velocity causes pressure is wrong (McLean, 2018). Thus, the Bernoulli-Based theory is not entirely complete and accurate.

The explanation in this theory is relatively similar to the "Equal Transit" theory. Both attribute lift production to a pressure difference between the top and bottom surface explained by Bernoulli's principle, although they differ in several assumptions. For this reason, many of the limitations of both theories are similar. This is because the Bernoulli-Based theory is also flawed in that Bernoulli's principle can only be applied in steady flowing, inviscid, and incompressible fluid. The theory also cannot explain how airplanes fly inverted or lifted by a symmetrical or flat wing with no thickness. Because for a pressure difference to form around the wing, the upper and lower surfaces of the wing cannot be equal. Furthermore, while this theory has several valid assumptions, there are still shortcomings that make the theory inaccurate in explaining lift by an airplane's wing.

Circulation Theory

One topic that has yet to be touched upon in the previous theories is the importance of vortices to lift production. First recognized by Frederick W. Lanchester and Ludwig Prandtl, who independently work theorized what is known as the Lanchester-Prandtl Wing Theory or Circulation Theory (Liu et al., 2015). The theory states that besides the laminar flow around the airfoil observed in other theories, there is also an additional circulatory flow. The formation of the circulatory flow is due to a starting vortex observed behind the airfoil when it moves through the air. This vortex is said to be produced by a Venturi tube formation on the top of the leading edge and below the trailing edge. The tube formation allows two high-speed airflows to occur, one atop the wing and a higher speed flow, which leaves the airfoil's trailing edge. When the two air flows connect at the trailing edge, airflow below the wing is described to curl up because of the speed difference between the two flows. Thus, forming the vortex through the Helmholtz theorem, which states that the formation of a vortex cannot exist alone and must be present with an anti-vortex. Through this theorem, the starting vortex creates an anti-vortex around the wing, also known as the circulatory flow (Singh et at., 2018).

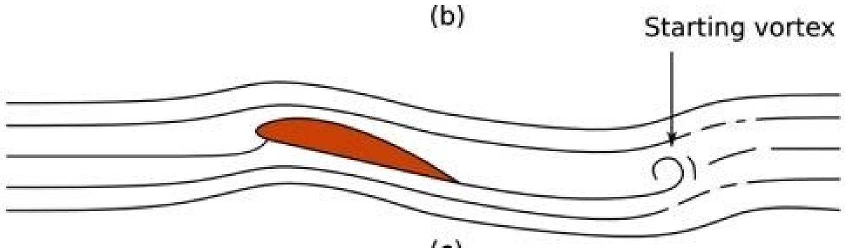

Figure 11. Formation of a starting vortex behind a wing. Figure taken from (Singh et at., 2018).

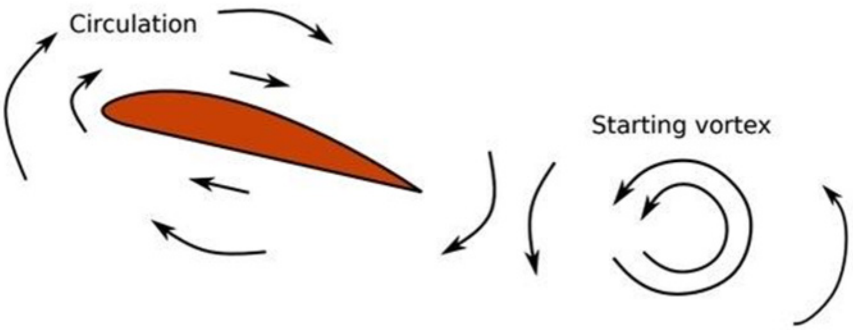

Figure 12. Vortex and anti-vortex pair generation due to Helmholtz theorem. Figure taken from (Singh et at., 2018).

From there, airflow following the Kutta-Joukowski theorem is denoted as the responsibility for lift generation. This is where significant misconceptions stem from, which will later be elaborated on. This theorem indicates that the flow will adjust itself to leave the trailing edge in a smooth flow (Chenyuan and Ziniu, 2014). To leave the trailing edge, the anti-vortex creates an increased flow speed that leaves the upper surface. The flow will continue to increase in speed until both streams, on top and below the wing, match up and leave the trailing edge satisfying the Kutta-Joukowski theorem. Thus, the anti-vortex results in higher speed above the wing and lower speed flow below. Then from Bernoulli's principle, the flow speeds also correspond with pressure (Singh et at., 2018). Higher speed flow above the wing leads to lower pressure, while lower speed flow below the wing leads to higher pressure. This pressure difference produces the aerodynamic lift in the direction of the weaker force.

The major shortcoming to this theory is that it does not provide a physical explanation for lift production, as the theorems used were misapplied. The theory mentions several theorems, which include the Venturi tube and effect, Helmholtz theorem, and Kutta-Joukowski theorem. However, the theory cannot be considered a qualitative physical explanation. As mentioned before, the implementation of the Venturi tube and its effect to explain how airflow accelerates over the wing is invalid. This is because an airfoil's upper surface is not a

Venturi tube, as there is no additional surface constricting the flow like that observed in a Venturi tube (Gonzalez, 2022). The Helmholtz theorem and Kutta-Joukowski theorem, used in the theory to explain the necessity of vortex and anti-vortex pair as well as induced circulation to lift generation, does not provide the desired non-technical approach. This is because the two theorems depend on advanced mathematical and aerodynamic theorems as opposed to direct physical arguments (McLean 2013). The theorems used are rooted in high-level math equations and formulas instead of principles that lift physically. While the theory can somewhat be physically explained, the use of the theorems would be misleading as they aim to provide formulas and equations to quantify lifting factors. Therefore, the different theorems in the theory are a limitation as they do not support a valid physical explanation for lift.

Another shortcoming to the theory is the one-way cause and effect that is explained to occur throughout the process of lift production by this theory. The theory implies that lift production is produced through several points that lead to the formation of the next. The explanation of the theory, although not directly stated, implies that lift is caused by the formation of a vortex which leads to an anti-vortex that causes circulation around the wing (Spalart, 1998). The theory is structured to describe lift in a list of steps that need to be completed to cause the following. However, this is misleading in that the starting vortex and the formation of a circulatory flow are seen more as byproducts of lift than what causes it. Moreover, the theory is misleading and somewhat inaccurate because while the theory is logically and mathematically correct, the progression of the argument does not demonstrate physical cause and effect.

A More Accurate Physical Theory of Lift

From the analysis of existing theories, it can be seen that every explanation has been limited in one way or another. Although explaining lift in itself is complicated, the mound of inaccurate theories poses a risk to the field of aerodynamics. Understanding how an airplane's wing generates the lift needed for flight is essential in aircraft design and safety. Thus, after extensive research, this section of this paper will provide a more accurate explanation for lift production by an airplane. This theory, proposed in McLean (2013), stands out as opposed to the others because it gives a more complete explanation for lift. One common shortcoming to the existing theories of lift is that they do not account for all necessary factors that contribute to and affect lift, making the theory incomplete. Therefore, the more accurate theory suggested is relatively longer and more complex to account for to avoid compromising correctness, accuracy, and completeness.

Airfoil Shape and Angle of Attack

Like all existing theories, the lift produced by an airplane will depend a lot on the shape of its wing and airfoil, the angle of attack, and the speed and density of airflow. All the listed factors are just a few aspects that cause and affect how much lift an aircraft will generate. An airfoil can be of any shape; as long as it is not too thick, the wing will be able to produce some lift at a compatible angle of attack. Depending on how an object will operate, different airfoil shapes can be used to produce the desired amount of lift in the situation. A cambered airfoil will produce more lift than a flat airfoil. Additionally, adding thickness to produce a rounded streamlined airfoil at the leading edge will generate a desired aerodynamic efficiency or lift-to-drag ratio (McLean 2013).

Lift Involves Newton's Third Law of Action and Reaction

Newton's third law states that every action has an equal and opposite reaction. This means that if an object exerts a force on another object, then the second object must exert an equal force in magnitude and opposite direction back at the first object. Like the momentum explanation, an instinctive way of visualizing lift is to imagine the angle of attack and airfoil shape cooperating to lead the airfoil to deflect the fluid, air, downwards as it flows past. The action of an airfoil deflecting incoming airflow downwards creates a reaction force that pushes upward back on the wing. As a result, lift is produced by the interaction in which the fluid and airfoil react and transfer equal and opposite forces (Ackroyd, 2015).

Lift Involves a Pressure Difference on the Airfoil Surfaces

As Bernoulli's principle explains, fluids of any type will always exert pressure. This means the pressure will push itself to any surface it comes in contact with. When an aircraft is at rest, with no wing and airfoil movement, the pressure stays the same at ambient pressure. The pressure pushing on the upper and lower surface are equal, and thus no lift or weight is generated. Then, when in flight, the fluid exerts lift force on the surfaces of an airfoil as a pressure difference. That is, the average pressure over the airfoil is always lower than ambient, while the average pressure below the airfoil is commonly higher than ambient. This pressure difference causes lift due to the higher pressure below pushing up on the wing (Batchelor, 2012). However, this pressure difference is relatively small, much less than the atmospheric pressure. The pressure on the upper surface also pushed down on the wing, while not at much as on the bottom surface. Therefore, to put into perspective, only about 0.2N of lift is produced per square centimeter of a wing. Thus, a large wing area is needed for lifting heavy and life-sized aircraft (McLean 2013).

Lift Involves Newton's Second Law of Force and Acceleration

To explain how pressure difference in the flow is maintained will require inquiring into the forces exerted on the air and the resulting accelerations of the air. Not just at the surface of the airfoil but in an extended region around the airfoil as well. The explanation goes as follows.

Airflows as if it were a continuous material that deforms to follow the shape of an airfoil. In the situation of an airfoil, the fluid that interacts with the wing is air. Air is composed of invisible gasses that are mass made up of many individual molecules. Unlike water molecules which are in constant contact with other molecules, air molecules frequently collide and move in rapid motion and directions. Due to the constant interaction between air molecules, the air flows like a continuous material. Illustrated in Figure 13, this allows the flow to deform and change to flow around obstructing objects like a wing.

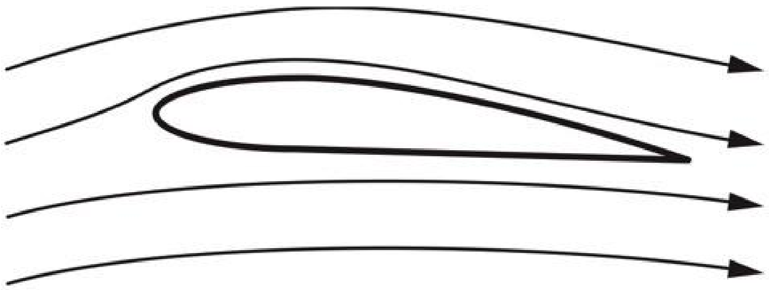

Figure 13. Airflow around an airfoil. Figure taken from (McLean 2013).

This leads to the second argument, in which the airfoil affects the direction and speed of the flow in a large area around the wing. Because deformation in the airflow occurs continually, the change in the direction of the flow is relatively slow. The speed and direction of this flow also vary over a large area. This is because of the airfoil's solid surface, which forces airflow to deform to its surface around the airfoil, known as a flow field. To generate lift, airflow is deflected downwards, as seen in the downward turning streamlines at the trailing edge of the airfoil in Figure 13 (Anderson & Eberhardt, 2010). However, to produce the downwards airflow, the airfoil must be cambered or obtain a positive angle of attack. While it may be evident that airflow is forced into being deflected downwards because of the airfoil shape, it is not only the flow near the airfoil's surface being affected. Priorly mentioned, the downwards airflow turns in a large area around the airfoil where the sloping is most significant near the airfoil and slowly subsides further away. Besides the airflow downwards turning, the pressure is also affected in a wide area called a pressure field; as illustrated in Figure 14, low-pressure forms above the airfoil while high-pressure forms below the airfoil. When air molecules exerting pressure touch the airfoil's surface, a pressure difference that produces lift is formed. The flow above the wing also accelerates, indicated by the longer arrow, while the flow below the wing decelerates (McLean 2013).

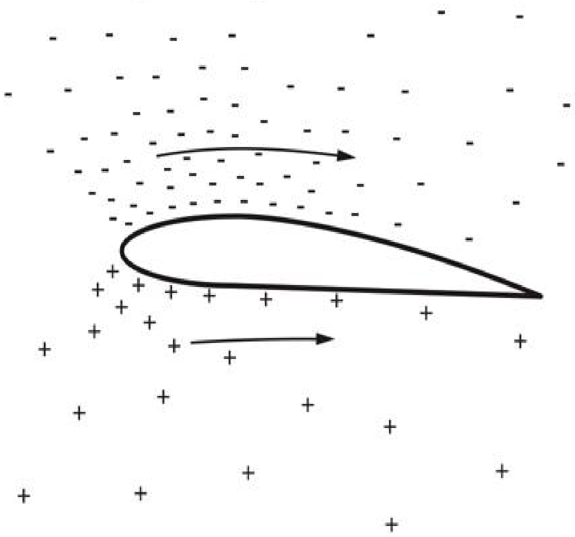

Figure 14. Pressure and velocity field around a lifting airfoil. Arrows illustrate flow acceleration. Plus signs indicate pressure higher than ambient while minus signs indicate pressure lower than ambient. Symbols placed closer together indicate a larger pressure difference. Figure taken from (McLean 2013).

The argument follows that the velocity and pressure field is sustained through a reciprocal cause and effect relationship. This relationship follows Newton's second law, which, in its simplest form, describes that a force applied to an object at rest causes it to accelerate in the direction of the force. Furthermore, the downwash, areas of high and low pressure, and changes in flow speed are all necessary factors that coincide in a reciprocal cause-and-effect relationship to produce lift. While the pressure difference above and below the airfoil generates lift, the downwards turning flow and flow speed changes sustain the pressure difference. Thus, these interactions produce and sustain lift (McLean 2013).

Effect of Camber, a Sharp Trailing Edge, and a Rounded Leading Edge

The airfoil's shape and its angle of attack affect the downwash of the airflow around an airfoil. Even a non-cambered or flat airfoil can produce downwards turning when placed at a particular angle of attack. Although introducing camber to an airfoil enhances the downwards sloping because of the curved surface. Therefore, cambered airfoils produce more lift, at a given angle of attack, than airfoils with no camber (Winslow et al., 2018).

The flow at the trailing edge also plays an essential role in controlling the downwash action. Figures 15 and 16 illustrate that the flow leaves the airfoil's trailing edge in a smooth stream. If this does not happen, the flow must turn around the trailing edge to the other surface. However, viscosity usually stops the flow from traveling around the edge. With viscosity, the trailing edge affects how the flow leaves the airfoil's trailing edge, which depicts the motion of downwards turning (McLean 2013). Thus, how the training edge directs the flow for downwash to occur, resulting in lift, is hugely impacted by the airfoil's camber and angle of attack.

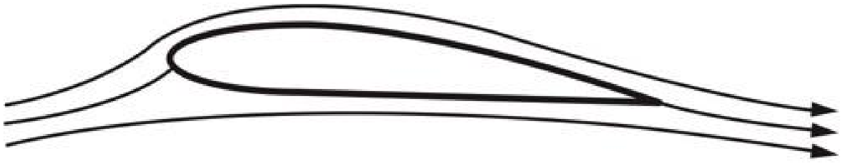

Figure 15. Streamline pattern at a low angle of attack. Figure taken from (McLean 2013).

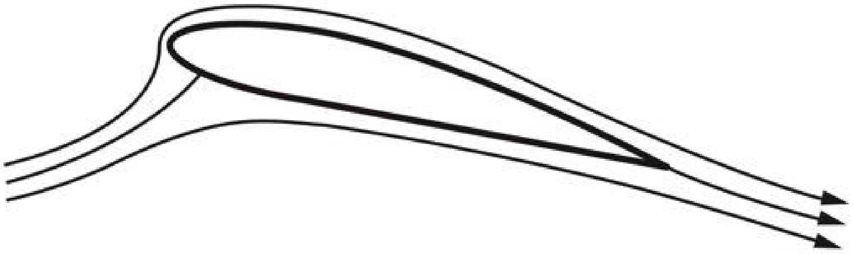

Figure 16. Streamline pattern at a medium angle of attack. Figure taken from (McLean 2013).

In contrast, the leading edge of an airfoil is rounded and thus very different from the trailing edge. Referring to Figures 15 and 16, the flow around the leading edge of an airfoil changes significantly when the angle of attack is changed. The upwash flow meets up almost at the airfoil's leading edge at a low angle of attack. However, at a raised angle of attack, the upward flow meets up with the airfoil from below and flows from the lower surface to the upper surface. Therefore, a wing at a higher angle of attack results in a larger obstacle for the flow at the leading edge. Thus, the leading edge directly affects the pressure and velocity field around the wing (McLean 2013).

Conclusion

The main aim of this paper was to explore how accurate existing theories of lift have been in explaining life by an airplane. To do so, several existing theories of lift were analyzed based on how accurate they have been in explaining lift by an airplane. In summary, the aerodynamic lift is not a simple concept to explain, hence the incorrect existing theories. Although believed and taught by many, the existing theories are still inaccurate and fail to explain the production of lift by an airplane correctly. The analysis of existing theories of lift reveals several common shortcomings and misconceptions. These misconceptions include not providing physical explanations for specific assumptions, misusing and misinterpreting principles and theorems, simplifying explanations thus not accounting for all necessary factors, proposing one-way causation relationships, confusing mathematical theories for qualitative physical theories, and conflicting experimental data. Because of these common shortcomings, the existing theories mentioned in this paper are incomplete and inaccurate. While the numerous lift explanations still inaccurately explain the concept, these discoveries have certainly contributed to a more concrete understanding of how an airplane's wing produces lift.

From there, a more accurate theory was presented with the research available. McLean (2013) proposed that accurately explaining lift production is not simple and involves various mechanisms. That is, lift involves Newton's second and third law of action and a pressure difference on the airfoil surfaces, explained through Bernoulli's principle. The theory also considers the importance of a wing's airfoil shape and angle of attack, the effect of camber, and the structure of a wing's leading and trailing edge. Moreover, the reason for perceiving this theory as accurate is because it is able to provide a complete explanation for life. One that accounts for all the factors affecting lift and provides proper physical explanations for the arguments. Although limitations to the suggested theory may still arise from advancing research, the suggested theory provides the most desirable and comprehensive explanation for lift. Furthermore, the suggested theory will be instrumental in the field of physics, especially aerodynamics and aeronautics.

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