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Axions as a Model of Dark Matter

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ABSTRACT

The true nature of dark matter is an extremely important and fundamental problem in the study of astrophysics, particle physics, cosmology and many other areas within the study of physics. This paper presents experimental evidence for the existence of dark matter through discussing the experimental results of mass profiling a galaxy and gravitational lensing. The fundamental properties of dark matter are then discussed, and evidence for these properties is presented. This allows further discussion of one of the most promising models of dark matter - the axion. The purpose of this paper is to present the evidence for the axion model, describe the nature of the theoretical axion particle, and to highlight the effects this model would have on other theories in physics such as solving the Strong CP Problem in the theory of quantum chromodynamics.

Introduction

The nature of dark matter is an extremely prominent and fundamental problem in many fields within physics such as astrophysics, particle physics and cosmology. According to studies, dark matter takes up 27% of the universe, significantly more than normal, observable matter, which takes up a mere 5% of the universe (Saleh, Alizadeh, & Dalili, 2020). This fact alone already makes ascertaining the true nature of dark matter extremely important. Furthermore, depending on the nature of dark matter, previously established fundamental physics theories such as the Standard Model and Quantum Chromodynamics may need to be altered leading to a greater understanding of cosmology as a whole. As such, researchers have been working to establish models for dark matter and testing them experimentally.

The biggest problem in building a working model for dark matter is how difficult it is to observe. The influence of its mass has been detected from experiments such as mass profiling and gravitational lensing. However, it doesn't interact with light, it cannot be visually observed, and it's very hard to detect its interactions with normal matter as it is almost collision-less. Despite these limitations, there are promising models for dark matter such as axions, WIMPs (Weakly Interacting Massive Particles), and primordial black holes (Bertone & Hooper, 2016). This paper will be discussing the axion dark matter model.

Axions are very small and very light bosonic particles that can be described as a classical field, similar to how photons can be described via an electromagnetic field. Axions have a few properties that make it a strong dark matter candidate (Garcia Irastorza, 2022).

First, they have a clear production mechanism. At the start of the universe, the axion field was frozen. As the universe began to cool, the graph of the potential of axions changed such that there were multiple potential minima. Oscillations around these potential minima are also known as particles.

Second, the existence of the axion solves the Strong CP Problem. It has been noted that both the strong and weak forces should violate CP symmetry. However, this violation has not been observed in the strong force. The axion field provides a clear the strong force to preserver CP symmetry.

The purpose of this paper is to describe the axion model, present the evidence for the axion particle being dark matter, and highlight the significance of this on other theories in the study of physics.

Evidence for Dark Matter

The existence of dark matter is widely accepted by the physics community due to overwhelming evidence for it. This section will discuss some of this evidence, and how it was discovered. One of these important pieces of evidence came from mass profiling a galaxy.

Measuring the mass enclosed by an orbit

In order to get the mass profile of a galaxy, we must first obtain an expression for the mass enclosed within a certain radius from the center of a galaxy. Using the formula for Newton's law of gravitation, and the formula for the centripetal force on an object in a circular motion, the following equation can be obtained.

$$\frac{mv^2}{r} = \frac{GM_{enc}(r)m}{r^2}$$
$$M_{enc}(r) = \frac{v^2 r}{G}$$

Where m is the mass of a test particle, v is its circular velocity, r is the radius from the center of the galaxy, G is the gravitational constant, and $M_{enc}(r)$ is the mass enclosed within r. Therefore, if the velocity and radius of an object orbiting the center of the galaxy can be obtained, the galaxy's mass profile can also be discerned (Zwicky, 2009).

Measuring the velocity of an object in orbit

The velocity of an object in orbit can be obtained via examining and comparing spectral lines. Atoms of specific elements will have distinct absorption and emission spectra resulting from the discrete energy drops between electron shells only allowing specific wavelengths of light to be absorbed/emitted. Using this knowledge, we can compare the spectral lines of an element such as hydrogen from a distant galaxy to the spectral lines of hydrogen in a lab. There will be a difference known as a red-shift/blue-shift, that can be used to calculate the velocity of the object that emitted the spectral line. This shift in the spectral lines arises due to the Doppler Effect.

The Doppler Effect affects the observed frequency of light. As the vast majority of galaxies are moving away from us, light observed from a distant galaxy will likely be red-shifted. This is because the relative speed of the distant galaxy causes an observed decrease in the frequency of light coming from it. The following formula allows the calculation of velocity from red-shift.

$$v = \frac{cf_{observer}}{f_{emitter} - f_{observer}}$$

Where c is the speed of light in a vacuum, $f_{observer}$ is the frequency of light as seen by an observer on earth, and $f_{emitter}$ is the frequency of light from the perspective of the galaxy that emitted the light (Zwicky, 2009).

The results from mass profiling a Galaxy

Before physicists used the mass enclosed within orbits of different radii to get the mass profile of a galaxy, the general consensus was that galaxies contained most of their mass in their center, where the vast majority of stars and planets lie as well as dilute interstellar gas composed mainly of hydrogen. There was expected to be a small amount of mass on the outer rims of the galaxy due to outlier stars and planets as well as smaller quantities of dilute interstellar gas but nothing significant. Hence, a graph of v^2 against r was plotted. From equation 1, if M is constant, then v^2 should

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decrease with radius until 0. However, as M is changing depending on r, physicists expected v^2 to rapidly increase near the center of the galaxy, to a maximum, then decrease to 0 as the $\frac{1}{r}$ term dominates the M term. This is because the mass was expected to increase rapidly at the center of the galaxy, then increase at a slower rate as r increased such that $\frac{dM}{dr}$ approaches 0 (Bertone & Hooper, 2016).

These experiments were first performed in 1970 (Rubin & Ford, 1970), by Vera Rubin, who took measurements from the Andromeda Galaxy, and plotted this graph of v^2 against r. Upon doing so, she found that v^2 did not decrease significantly as expected but instead stayed significant even with large values of r. She repeated this experiment in 1978 (Rubin, Ford & Thonnard, 1978) with more galaxies and obtained the same result. These surprising results were also supported by Fritz Zwicky's experiment in 1933 (Zwicky, 2009), who did the same experiment with galaxy clusters as opposed to singular galaxies. As such, this implies that the M term was still quite dominant and was increasing at a significant rate far out in the galaxy. The only explanation for this is that there is some matter with a significant mass in the outer regions of galaxies - dark matter.

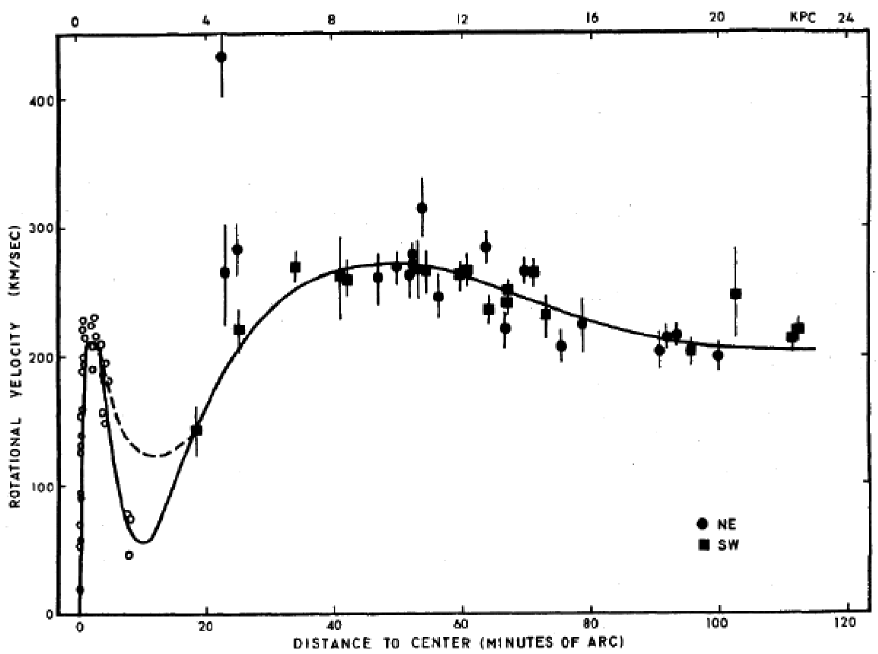

Figure 1. Rotation Curve of the Andromeda Galaxy as done by Vera Rubin (1970) (Rubin & Ford, 1970)

Properties of Dark Matter

Empirically, we know that dark matter has 4 important properties.

1. It is dark, meaning that it does not interact with electromagnetic radiation.
2. It behaves like matter (as demonstrated from the results of mass profiling a galaxy).
3. It is cold, meaning that dark matter particles have kinetic energies much smaller than their mass energies.
4. It is collision-less, meaning that it doesn't interact very strongly with normal matter or itself.

The fact that dark matter is dark is fairly obvious. If it wasn't dark, then we would be able to see it. The fact that dark matter behaves like matter is also obvious, as it is for this reason that physicists postulated the existence of dark matter to begin with. However, the other two properties are more difficult to discern.

How we know that Dark Matter is cold

One piece of evidence for dark matter being cold is that if it were hot, it would have enough energy to escape the galaxy in which it resides. The escape velocity for a galaxy is given by the following formula:

$$v_{escape}^2 = \frac{2GM}{r}$$

Upon substituting the values of mass and radii for a variety of galaxies, it was found that the average escape velocity for a galaxy is significantly less than the speed of light. Because there is still a large amount of dark matter within galaxies, the speed of dark matter should also be much less than the speed of light. As such, on a natural scale, dark matter is cold.

Another piece of evidence for dark matter being cold is that it had to be cold in order to collapse and form structures like galaxies and galaxy clusters. If we assume that dark matter has a spherical distribution and consider the force on an object with mass m that lies at the edge of said sphere, the following equation can be obtained.

$$F = \frac{-GMm}{r^2} = ma$$

Where F is the force experience by mass m.

$$a = \frac{-GM}{r^2} = \frac{-G\rho_{dark\ matter}V}{r^2} \approx \frac{-G\rho_{dark\ matter}r^3}{r^2} = G\rho r = \frac{dv}{dt} \approx \frac{\Delta v_m}{\Delta t_{free\ fall}}$$

Where $t_{free\ fall}$ is the time taken for mass m to reach the center of the spherical distribution of mass and v_m is the velocity of mass m

$$G\rho r t_{free\ fall} = v_m$$

$$G\rho r t_{free\ fall}^2 \approx r_{sphere}$$

$$t_{free\ fall} \approx \frac{1}{\sqrt{G\rho}}$$

Since we know that dark matter collapses and forms structures, $v_{initial} \lessapprox r\sqrt{G\rho}$.

By substituting values of density and radius of galaxies into the inequality above, it is found that the velocity of dark matter is much less than the speed of light, meaning that dark matter must be cold. It should be noted that the expansion of the universe was not taken into account. However, even when taking this into account, the same conclusion is reached (Spergel, & Steinhardt, 2000).

How we know that Dark Matter is collision-less

From Einstein's Theory of General Relativity, we know that gravity can cause light to bend, hence a large enough mass in space can act as a lens for light, bending it and making it appear as if it came from somewhere else. As such, we can use this phenomenon to determine the mass distribution of a large object such as a galaxy cluster by comparing the strength of gravitational lensing it causes in different regions.

When observing the Bullet Cluster, a galaxy cluster that consists of 2 colliding galaxies, examining the behavior of the galaxies' constituents gives interesting results. The planets and stars in the galaxies almost never collide with each other due to their small size relative to the distance between them. Conversely, the large amount of hydrogen gas in the galaxies does collide with itself, as the distance between hydrogen atoms is very small. The mass of hydrogen gas in a galaxy is greater than the mass of its planets and suns, hence without taking dark matter into account, gravitational lensing should be strongest in the center of the galaxy cluster due to the collisions between the hydrogen atoms. However, gravitational lensing is actually strongest at the edges of the cluster, where the planets and stars lie. This suggests that something of a very significant mass lies on the edges of the bullet cluster - dark matter. This implies that dark matter is collision-less, because if it collided with itself, then the gravitational lensing would still be strongest in the center of the galaxy cluster, with both hydrogen gas as well as dark matter being there (Clowe et al., 2006).

Axions

Now that the properties of dark matter and the evidence for them have been discussed, we can now discuss the possible models for dark matter. Although we currently are uncertain of what exactly dark matter is, there are some very strong models for it, one of them being the axion. An axion is a theoretical particle that has a very small mass (much less than 1eV), 0 charge, and 0 spin, making it a boson. Axions also are pseudo-scalars, which means that they act as scalars that flip sign under a parity inversion. A parity inversion refers to the flip in the sign of a spatial coordinate and is effectively a reflection.

Mass range for bosonic dark matter

Because axions are a bosonic model of dark matter, we can find its mass range. Bosons have a special property, they do not have to obey Pauli's exclusion principle. This means that they can "stack on top of each other". In other words, it is possible for there to be more than 1 boson per de Broglie Volume.

Every particle has a wavelength associated with it known as the de Broglie wavelength.

$$\lambda_{de\ Brogile} \approx \frac{1}{mv}$$

Consider a volume of $\lambda^3_{de\ Broglie}$. This is known as the de Broglie Volume.

The maximum possible density of a non-relativistic fermion field is one particle per de Broglie Volume. We can use this to define a mass range for which a particle must be a boson if it is dark matter. Consider a scenario in which there is one particle per de Broglie Volume. To model this scenario, we can use the following expression.

$$\rho_{dark\ matter} \approx m_{dark\ matter} n_{dark\ matter}$$

where $n_{dark\ matter}$ is the number of dark matter particles within a given volume, $m_{dark\ matter}$ is the mass of said dark matter, and $\rho_{dark\ matter}$ is its density. This expression can be rearranged to obtain:

$$n_{dark\ matter} \approx \frac{\rho_{dark\ matter}}{m_{dark\ matter}}.$$

The following series of expressions can now be written to postulate the mass range for bosonic dark matter.

$$n_{axion\,sin\,deBrogileVolume} \approx n_{darkmatter}\lambda^3_{deBroglie} \approx \frac{\rho_{darkmatter}}{m^4_{darkmatter}v^3} < 1$$

Where $n_{axions\ in\ de\ Broglie\ Volume}$ is the number of axions in the de Broglie Volume, v is the velocity of dark matter, and $\lambda_{de\ Broglie}$ is the de Broglie wavelength.

$$m_{dark\ matter} > \left(\frac{\rho_{dark\ matter}}{v^3}\right)^{\frac{1}{4}}$$

$$\rho_{darkmatter} \approx \frac{0.3GeV}{cm^3}$$

(Note: this is the local dark matter density in the milky way (Brézin, 2021)) Using the mass of the milky way of approximately 10^{12} solar masses, and the radius of the milky way of approximately 10^5 light years, we find that the virial velocity of local dark matter ≈ $10^{-3}c$

$$v_{dark\ matter} \approx 10^{-3}$$

Where $v_{dark\ matter}$ is the velocity of a dark matter particle

$$\therefore m_{fermionic\ dark\ matter} \geq 7eV$$

Where $m_{fermionic\ dark\ matter}$ is the mass of a fermionic dark matter particle

$$\therefore m_{bosonic\ dark\ matter} < 7eV$$

A more sophisticated calculation was performed in 1979 by Scott Tremaine, and James E. Gunn (Tremaine & Gunn, 1979). In their paper they calculated that mass range for which dark matter must be bosonic is:

$$\therefore m_{bosonic\ dark\ matter} < 101eV$$

As such, any particle with mass more than 7eV can either be a boson or fermion, but any particle with mass less than approximately 7eV must be a boson. For this reason, we can describe axions as a classical field (Peccei & Quinn, 1977).

Scalar Field Dark Matter

In order to better describe the axion, we should first consider an expression for the Newtonian behavior of normal matter. Newton's second law states that $F = ma$ where F is force, m is the mass of the object that experiences this force, and a is the acceleration of mass m as a result of this force. Since a is the second time derivative of displacement, we can rewrite a as $\ddot{x}$, where x is the degree of freedom of mass m. We can further consider the potential of this Newtonian object. It is known that $F = -\frac{dV}{dx}$, where V is the potential of the object with mass m. With this knowledge, the following expression can be obtained.

$$m\ddot{x} + \frac{dV}{dx} = 0$$

$$\ddot{x} + \frac{dV}{dx}\frac{1}{m} = 0$$

This is a second order differential equation. Because Newtonian physics works in this way, a reasonable place to begin creating a theory for the axion would also be with a second order differential equation. Consider the following second order differential equation model for an axion.

$$\partial_t^2\phi + \frac{\partial V(\phi)}{\partial\phi} = 0$$

Where ϕ is the degree of freedom of the axion, ∂_t^2 is the second time derivative of ϕ, and $V(\phi)$ is some function of ϕ that gives the value of potential.

This is the simplest possible description of the axion. However, there is a clear problem with this expression - it is not Lorentz invariant. Einstein's famous mass energy equation states that $E^2 = m^2 + p^2$, where E is energy, m is mass, and p is momentum. We know that mass is Lorentz invariant, hence $E^2 - p^2$ must also be Lorentz invariant. Hence, in order to make the previous expression Lorentz invariant, another term must be added.

$$\partial_t^2\phi - \partial_x^2\phi + \frac{\partial V(\phi)}{\partial\phi} = 0$$

Where $\partial_x^2\phi$ is the second space derivative of ϕ. This term is added because energy is closely related to changes in time, while momentum is closely related to changes in space. For simplicity's sake, ∂_x will be expressed as ∇.
Using this to rewrite the previous expression gives:

$$\partial_t^2\phi - \nabla^2\phi + \frac{\partial V(\phi)}{\partial\phi} = 0$$

It turns out that the above expression in a flat space-time is very accurate. However, we know from Einstein's theory of General Relativity that this is not the case. In an expanding spacetime, the above expression becomes:

Equation 1: Scalar field dark matter equation

$$\partial_t^2\phi + 3H(t)\partial_t\phi - \frac{\nabla^2}{a^2}\phi + \frac{\partial V(\phi)}{\partial\phi} = 0$$

Where a is the scale factor of the universe, and H is the Hubble parameter, defined as $\frac{\dot{a}}{a}$ (De Jesus, Pereira, Malatrasi & Oliveira, 2016).

Proof checking the scalar field equation for an axion

The expression found in the previous section seems to describe axions quite accurately. However some checks must be performed before it can be certain that this scalar field behaves like dark matter. The most important thing to check is whether this expression allows axions to behave like matter - an important property of dark matter. To verify this, we can check whether the energy density of axions that behave as described by the previous expression dilutes correctly as the universe expands.

Consider an arbitrary amount of axions within a box of finite space. The energy density of this box can be written as $\rho = \frac{E_{box}}{V_{box}}$. Consider a scenario in which this box expands such that each of its dimensions increases with scale factor a. In this scenario, matter dilutes as $\rho \propto \frac{1}{a^3}$. Hence, because dark matter behaves like normal matter, an expression for the energy density of dark matter should be found in order to check if it also dilutes as $\rho \propto \frac{1}{a^3}$. The energy density of ϕ is equal to its kinetic energy plus its potential energy.

$$Kinetic\ Energy \sim \frac{1}{2}mv^2 \sim \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}(\nabla\phi)^2$$

$$Potential\ Energy \sim V(\phi)$$

$$\rho_\phi \sim \frac{1}{2}mv^2 \sim \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}(\nabla\phi)^2 + V(\phi)$$

Where ρ_ϕ is the energy density of dark matter.

Consider the homogeneous part of Equation 1 where there is no spatial derivative term.

Equation 2: Homogenous part of Equation 1

$$\partial_t^2\phi + 3H(t)\partial_t\phi + \frac{\partial V(\phi)}{\partial\phi} = 0$$

Let $V(\phi)$ be some arbitrary function of ϕ that can be Taylor expanded to form some polynomial with arbitrary coefficients.

Equation 3: Equation for the potential of ϕ

$$V(\phi) \sim c_0 + c_1\phi + c_2\phi^2 + c_3\phi^3\ldots$$

At small values of ϕ, the higher order polynomial terms are negligible. Furthermore, if we consider the potential at a minimum, there is a stationary point. As such, the first derivative of the potential is 0, hence c_1 is also 0. Hence we only have to consider the order 2 term.

Equation 4: Order 2 term for the potential of ϕ

$$\frac{\partial V(\phi)}{\partial\phi} \sim m^2\phi$$

Later, it will become evident that the equation for $V(\phi)$ is very similar to the simple harmonic oscillator equation, where c_2 is the coefficient that represents the natural oscillation frequency of the ϕ degree of freedom. As oscillation frequency is closely related to energy, and most of an axion's energy comes from its mass, let c_2 be $\frac{1}{2}m^2$, where $\frac{1}{2}$ is an arbitrary coefficient used to remove the factor of 2 on the right-hand side of Equation 4. Substituting Equation 4 into Equation 2 gives:

Equation 5: Equation obtained from substituting Equation 4 into Equation 2

$$\partial_t^2\phi + 3H(t)\partial_t\phi + m^2\phi = 0$$

Because this expression looks similar to the expression for a simple harmonic oscillator, a sensible guess for a solution for ϕ is in the form $\sigma e^{i\omega t}$. Where ω is an angular velocity, and σ is some coefficient. Substituting this form into Expression 5 and evaluating respective time derivatives gives the following:

$$(-\omega^2 + 3\omega i H(t) + m^2)(\sigma e^{i\omega t}) = 0$$

Upon using the quadratic formula on the left-hand term, the following expression can be obtained

$$\omega = i\frac{3H(t)}{2} \pm \sqrt{m^2 - \frac{9H(t)}{4}}$$

When $H \ll m$ in late times in the universe, $\omega \approx \pm m + i\frac{3H(t)}{2}$. Substituting this into the original guess for the solution of ϕ, $\sigma e^{i\omega t}$ gives the following:

$$\sigma \sim \sigma_0 e^{-\int_{t_0}^{t}\frac{3H(t)}{2}dt}$$

$H(t)$ must be written in integral form as it is a function of time. Since H(t) is defined as $\frac{\dot{a}}{a}$:

$$\sigma \sim \sigma_0 exp[\int_{t_0}^{t}\frac{3}{2a}\frac{da}{dt}dt]$$
$$\sigma \sim \sigma_0 exp[\int_{a_0}^{a}\frac{3}{2a}da]$$
$$\sigma \sim \sigma_0 exp[\frac{-3}{2}ln(\frac{a}{a_0})]$$
$$\sigma \sim \sigma_0(\frac{a}{a_0})^{-\frac{3}{2}}$$
$$V(\phi) \sim \frac{1}{2}m^2\phi^2 \sim \frac{1}{2}m^2\sigma^2$$
$$V(\phi) \sim \frac{1}{2}m^2\sigma^2(\frac{a}{a_0})^{-3}$$

From the above expression it can be seen that $V(\phi) \propto \frac{1}{a^3}$. Hence $\rho_\phi \propto \frac{1}{a^3}$.

It should be noted that the reason that kinetic term was not discussed is because the oscillation of the degree ϕ term was at its amplitude. At this point, there is no kinetic energy, only potential.

Scalar Field Dark Matter Initial Conditions

For axions to exist, there must be some sort of excitation of a quantum field. We can describe an axion as an excitation around its ground state, where its potential $V(\phi)$ is a minima. Due to this excitation, the field should oscillate around a ground state. These oscillations are axions. However, for an axion field to have any sort of oscillatory motion, there must have been an initial displacement, otherwise the potential would have stayed at zero and never moved, hence never resulting in the creation of axions. This initial excitation can be explained due to the state of the axion field at early times.

In the beginning of the universe, physicists believe that there was a period of accelerated expansion known as inflation. At this time, the Hubble rate behaved as a constant, hence the Hubble rate at this time will be referred to as H_I.

$$H_I = \frac{\dot{a}}{a}$$
$$\dot{a} = H_I a$$

Because the derivative of a is a multiple of a, $a = e^{H_I t}$.

As the Hubble rate is associated with the magnitude of damping in the scalar field equation for an axion, and since a is increasing exponentially, any motion in an axion's degree of freedom, ϕ is quickly damped away.
Consider the angular frequency of the Scalar Dark Matter field. Solving Equation 5 gives:

$$\omega = i\frac{3H_I}{2} \pm \sqrt{m^2 - \frac{9H_I^2}{4}}$$

Consider the case where $H_I \gg m$. In this case the two solutions will be:

$$\omega = i3H_I$$
$$\omega \approx \frac{im^2}{3H_I} + \mathit{Higher\ order\ terms\ of\ Taylor\ Expansion}$$

Both of these solutions only contain an imaginary component, meaning that there is no oscillatory motion, only damping. The first solution - $i3H_I$ is large, hence almost all energy will be damped away very quickly. The second solution - $\frac{im^2}{3H_I}$ is very small, hence damping is very slow, meaning that the axion field is "frozen". As such, there is no reason to believe that the axion initial position must have been at its minimum. This explains how there could have been some initial displacement that caused the axion field to start oscillating (Garcia Irastorza, 2022).

Spontaneous Symmetry Breaking

The potential of an axion, $V(\phi)$, is determined by its interactions with other particles, so it heavily depends on its background. At the start of the universe, there were a lot of very hot particles. Hence consider an equation for the potential of a field taking into account a thermal bath.

$$V(\phi) = -\mu^2\phi^2 + \lambda\phi^4 + m_{thermal\ bath}^2\phi^2 + \mathit{Higher\ order\ even\ polynomial\ terms}$$

Where μ and λ are arbitrary coefficients in the expression for $V(\phi)$ without a thermal bath, and $m_{thermal\ bath}$ is the mass of the particles in the thermal bath. In this case, because the potential function is symmetric, $V(\phi)$ must be an even function, so there can only even polynomial terms.

This equation assumes that $m_{thermal\ bath}^2$ must be positive to avoid an infinite potential runoff.

Typical thermal field calculations give $m_{thermal\ bath}^2 \sim g^4T^2$, where g is some coupling constant and T is the average temperature of the thermal bath. This is because in terms of energy, $V(\phi)$ is order 4, which means all its terms must also be order 4. The first two terms with coefficients of μ and λ can't be associated with the temperature of the thermal bath as they remain in the expression without the thermal bath. We also know that coefficients of polynomial terms of order 4 or above cannot represent the temperature of the thermal bath, because in order to remain as an order 4 term, they must be constants or negative polynomial terms. While these constants and negative polynomial terms may vary with temperature, and may even be significant at very high energies, at lower energies they will be far less significant than the $m_{thermal\ bath}$ term, as it increases linearly with temperature. As such, the $m^2_{thermal\ bath}$ is the only term that should be associated with temperature. In this case, $m^2_{thermal\ bath}$ can be any arbitrarily named coefficient, but it is being represented as a mass because we know that mass and temperature are both order 1 in terms of energy.

At high temperatures, $m^2_{thermal\ bath}\phi^2$ will be dominating term. Since this is order 2 in terms of ϕ, the function $V(\phi)$ will resemble a quadratic.

In this scenario, the axion only has one degree of freedom, hence $V(\phi)$ only has one minima. However once the temperature of the surrounding particles decreased past a certain temperature, $V(\phi)$ starts to resemble a higher order even polynomial which has more than one minima. As axions initially used to be at rest at the original minima, after $V(\phi)$ changes, axions will spontaneously fall into one of the new minima.

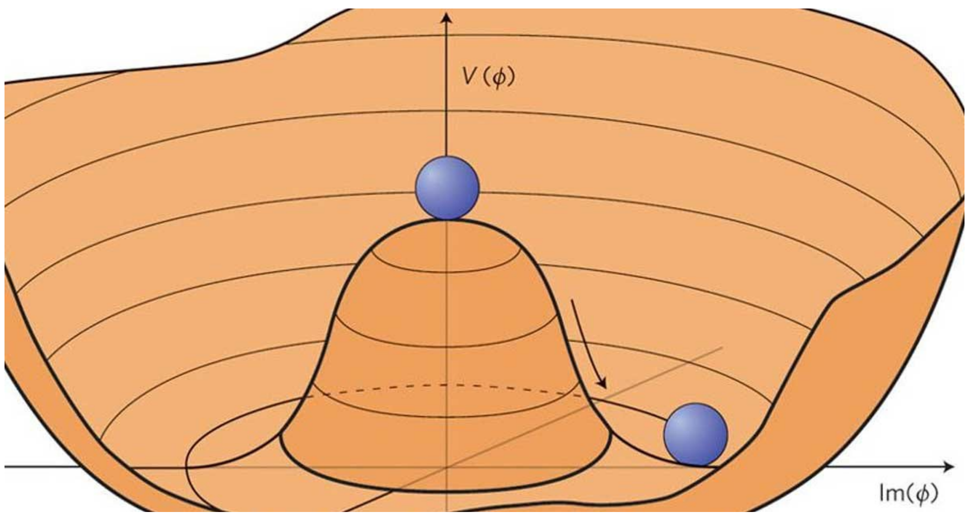

Figure 2. Diagram of the potential of an axion (Brézin, 2021)

Now consider a two-dimensional example, where there are 2 degrees of freedom. To visualize this, consider the axion to be at rest at the bottom of a "bowl" shaped object. In this case, when the temperature of the thermal bath decreases past a certain critical temperature, the axion will now lie at the top of a "hill", or "Mexican Hat", that slopes downwards in all directions, with a ring of minima below it (See Figure 2). Once an axion falls into one of these minima, its position can now be described by some angle θ along the circle (Garcia Irastorza, 2022).

The Strong CP Problem

One reason why axions are favoured as one of the most plausible dark matter models is that it conveniently solves another persistent problem in the world of physics - the Strong CP Problem

When discussing the interactions of a particle, we can write in the form of $V(\phi_1, \phi_2, \phi_3 \ldots)$, where ϕ_1, ϕ_2, ϕ_3 etc. are parameters. Another way to do this via a Lagrangian. In Newtonian mechanics this is expressed simply as kinetic energy minus potential energy. One example of this is the QCD (Quantum Chromodynamics) Lagrangian. This Lagrangian contains all the interactions due to the strong force between quarks and gluons.

One of the terms in this Lagrangian is:

$$L \supset \theta \frac{g^2}{32\pi^2} G^{\mu\nu} \tilde{G}_{\mu\nu}$$

The interesting thing about this term in the Lagrangian is that it violates CP symmetry (Bertone & Hooper, 2016). There are 3 fundamental discrete things that a particle can conserve: Charge, Parity, and Time. This means when one of these properties is inverted, the particle should stay the same. Under an inversion of all 3 at the same time, the particle should stay the same. In some interactions, some of these symmetries are allowed to violated. For example, in both the strong and weak forces, a violation of CP symmetry is allowed (asymmetric upon both a charge and party inversion) as seen from the term in the QCD Lagrangian. However, while this violation has been observed in the weak force, it has not been meaningfully observed in the strong force. Experiments to detect it found that the θ parameter in the term in the QCD Lagrangian that violates CP symmetry must be of order magnitude less than 10^{-9} (Bertone & Hooper, 2016).

The fact that this parameter is so close to zero, where it could have been any number makes it seem like a “tuning", as if it was somehow forced into being closed to zero. The probability that happens to be so close to zero as opposed to any other higher order of magnitude number makes it seems like its value isn’t a coincidence. The small value of θ would make sense if it was a function that could be minimized, as it would naturally tend towards zero. However, θ in the QCD Lagrangian is simply a constant and cannot change (Wilczek, 1978).

The Axion Field

In 1977, Roberto Peccei and Helen Quinn (Peccei & Quinn, 1977) proposed a solution to this “tuning" problem - a new pseudo scalar field $\tilde{\theta}$ to couple to gluons. With this new field added to the QCD Lagrangian, the previous θ term becomes $\theta + < \tilde{\theta} >$. This now provides an elegant solution to the CP problem. θ can now be any number, and doesn’t necessarily have to be close to zero because the new $\tilde{\theta}$, when minimized, will cancel the θ term out, resulting in a close to zero term. This new $\tilde{\theta}$ field represents the axion’s position along a circle of potential minima as discussed in the previous section (Garcia Irastorza, 2022).

Conclusions

In this work, we discussed how mass profiling a galaxy provided strong evidence for the existence of dark matter. We also discussed its properties and how they were discerned in order to explain one of the most promising models for dark matter: axion. This work discussed the mass range for axions, and hence explained why axions can be described as a classical field. An expression for scalar dark matter was then postulated, then subsequently checked to see if the behaviour it describes aligns with the properties of dark matter. Furthermore, the production mechanism of axion particles via spontaneous symmetry breaking of the graph of the potential of the axion was discussed. Finally, the possibility of the Peccei–Quinn axion field was discussed to solve the Strong CP Problem.

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