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Warping Spacetime: When Theoretical Physics Meets Science Fiction

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ABSTRACT

Warp drives, though first envisioned in science fiction, have become a topic of scientific interest within theoretical physics. Miguel Alcubierre was the first one to propose a warp drive spacetime in 1994. His geometry is, however, considered unphysical: it violates all energy conditions and requires large amounts of exotic matter. Several modifications to his metric have been proposed which reduce these energy requirements, but it is still unlikely that warp drives will ever be technologically useful. Even so, they make for very interesting thought experiments, forcing physicists to face the limits of their current theories. Here we will discuss the Alcubierre drive, its energy condition violations, and its energy requirements as obtained from the quantum inequalities. The general definition of superluminal travel is briefly discussed. We also study the horizon problem and the possibility of closed timelike curves within this spacetime. Some recent progress in warp drive research is presented, followed by a conclusion which expands on the physical plausibility and research interest in warp drives.

Introduction

As we know, Einstein's theory of general relativity (GR) relates the stress-energy to the curvature of spacetime through Einstein's field equations:

Equation 1: Einstein's field equations

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi GT_{\mu\nu}$$

where $R_{\mu\nu}$ is the Ricci tensor, R is the Ricci scalar, G is the gravitational constant, and $T_{\mu\nu}$ is the stress-energy tensor.

Also recall that it can be useful to classify spacetime and stress-energy according to which "energy conditions" they do or do not respect. Among the most important conditions are:

1. The Weak Energy Condition (WEC), which states that $T_{\mu\nu}t^{\mu}t^{\nu} \leq 0$ for all timelike vectors t^{μ}; equivalently, $\rho \geq 0$ and $\rho + p \geq 0$; physically, the WEC means that local energy density is positive
2. The Null Energy Condition (NEC), which states that $T_{\mu\nu}l^{\mu}l^{\nu} \geq 0$ for all null vectors l^{μ}; equivalently, $\rho + p \geq 0$

If you need a more thorough review of general relativity, refer to Price (1982), Hartle (2003), or Carroll (2004).

In GR, there are two main ways to arrive at a solution. The first one is to start with a given stress-energy tensor, generally coming from a known matter or energy source, and to use the field equations to find the consequential curvature of spacetime. The second and less conventional one is to tailor a metric for a specific purpose and use the field equations to find the stress-energy required to form such a metric. These solutions are sometimes called reverse-engineered solutions. Warp drives fall into this second category.

The use of warp drives was first envisioned in science fiction and popularized by Star Trek. Miguel Alcubierre discovered, in 1994, a metric that resembled such "sci-fi" objects and could allow for superluminal travel (Alcubierre, 1994). His warp drive makes faster-than-light travel possible by a local expansion of the spacetime behind a spaceship and a local contraction of the spacetime in front of it. As we will see, his geometry is generally considered unphysical because of the large amount of exotic matter (matter that violates the energy conditions listed above) it requires (Lobo, 2017). It is worth noting that this is a fairly common feature among the reverse-engineered solutions mentioned before (including wormholes).

This article aims to evaluate the physical plausibility of the Alcubierre drive and of warp drives in general, as well as to understand the importance of warp drive research, regardless of such plausibility. We additionally wish to present warp drives in a manner accessible to beginning general relativity students.

To that end, we start by reviewing Alcubierre's metric, as well as its potential use in superluminal travel. We then spend some time understanding the stress-energy requirements for such a metric. Next, we analyze some interesting challenges present in his solution, such as the horizon problem and the possibility of closed timelike curves. Finally, we present some recent progress in warp drive research and conclude.

Alcubierre Metric

In Cartesian coordinates and natural units ($G = c = 1$), the Alcubierre metric is (Lobo, 2017)

Equation 2: Alcubierre metric in Cartesian coordinates

$$ds^2 = -dt^2 + dx^2 + dy^2 + (dz - v(t)f(x, y, z - z_0(t))dt)^2$$

where $z_0(t)$ is the position of the warp bubble's center, and $v(t) = \frac{dz_0(t)}{dt}$ is the bubble's velocity, moving along the $+z$ axis.

The form function $f(x, y, z)$ should obey the general property of having value 0 outside of the bubble and 1 inside of it. This means that, outside of the bubble, the metric becomes

Equation 3: metric outside of the warp bubble

$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2$$

which is the metric for flat spacetime (the spacetime metric of special relativity).
We may rewrite Equation 2 this in the 3+1 formalism of general relativity as (Alcubierre, 1994)

$$ds^2 = -(\alpha^2 - \beta_i \beta^i)dt^2 + 2\beta_i dx^i dt + \gamma_{ij} dx^i dx^j$$

where

$$\alpha = 1,$$
$$\beta^z = -v(t)f(r),$$
$$\beta^x = \beta^y = 0,$$
$$\gamma_{ij} = \delta_{ij}$$

Alcubierre chose the specific class of spherically symmetric form functions $f(r)$, where $r = \sqrt{x^2 + y^2 + z^2}$. We consider the form function

Equation 4: form function considered

$$f(r) = \frac{\tanh \phi(r + R) - \tanh \phi(r - R)}{2 \tanh \phi R}$$

where $R, \phi > 0$, R is the radius of the bubble, and ϕ is inversely proportional to the thickness of the bubble's wall. Note that, with a sufficiently large ϕ (a sufficiently small bubble wall thickness), the form function rapidly approaches a top hat function: $f(r) = 1$ for $r \in [0, R]$ and $f(r) = 0$ for $r \in [R, \ \infty]$.

Considering a spaceship placed within the warp bubble, the expansion of the volume element is (Lobo, 2017)

Equation 5: volume element expansion

$$\theta = U^{\mu}_{;\mu} = v \frac{\partial f}{\partial z} = v \frac{z - z_0}{r} \frac{df(r)}{dr}$$

We can plot this volume expansion for $v = R = 1$, $z_0 = 0$, and $\alpha = 8$, as shown in Figure 1. The result is as expected: negative values in the front region, and positive values in the back. In other words, a volume contraction ahead of the centered spaceship, and a volume expansion behind it.

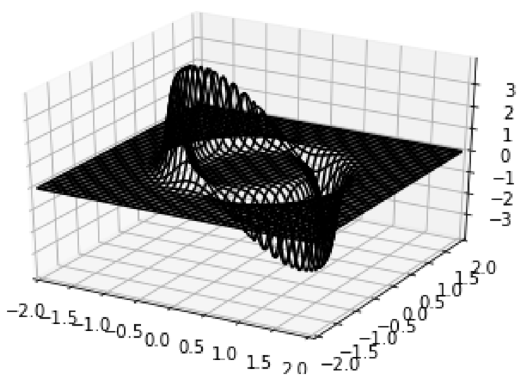

Figure 1. Volume element plot for the Alcubierre warp drive.

By changing the values of the parameters chosen, we can obtain modified volume element plots. In Figure 2, $\alpha = 4$. In Figure 3, $R = 2$. In Figure 4, $v = 2$. We only change one parameter at a time, keeping all other values as they were in Figure 1.

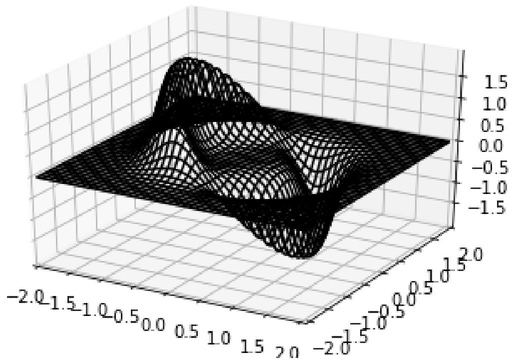

Figure 2. Modified volume element plot with smaller α (larger bubble wall thickness).

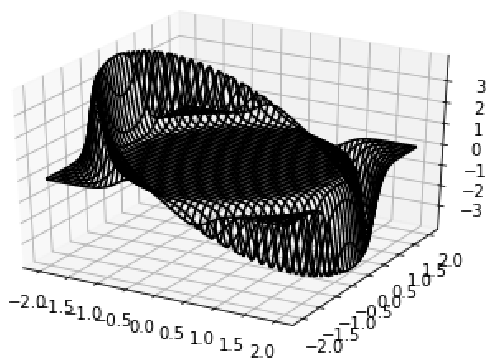

Figure 3. Modified volume element plot with larger radius.

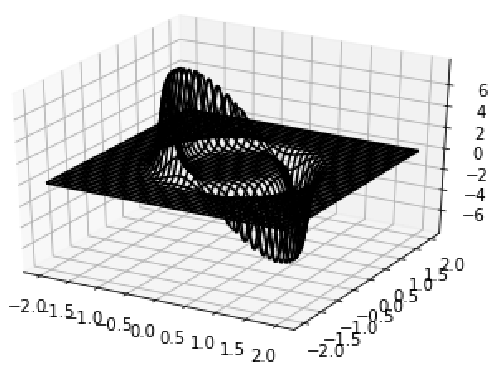

Figure 4. Modified volume element plot with larger velocity.

Faster-than-light Travel

Now that we understand the mathematical and geometrical properties of Alcubierre's metric, we devote this section to an exploration of how it might allow for superluminal, or faster-than-light, travel.

Meaning and Definition

To understand what it means for a particular solution to allow for faster-than-light travel, we need to understand what faster-than-light (FTL) travel is. It might seem simple: FTL travel is travel at a speed greater than that of light. However, we must remember that we are dealing with a general relativity background. Therefore, we need to consider the impact of different reference frames and coordinate choices.

There has been a lot of work done regarding the general definition of superluminal travel (Olum, 1998; Visser, Bassett & Liberati, 2000; Visser, Bassett & Liberati, 1999). One of the main obstacles found was that of metrics which seem to imply FTL travel but do not. Olum showed a particular example of this where a flat spacetime metric expressed in an unusual choice of coordinates seems to allow for superluminal travel. He also proposed a general definition according to which superluminal travel requires that the path to be traveled reach a destination surface earlier than any neighboring path (Olum, 1998). It is good to keep in mind that, while the term FTL travel is not formally defined here, there exist precise definitions which hold for the case of the Alcubierre drive.

Example

We now walk through a quick demonstration of how the Alcubierre drive might allow for superluminal travel (Lobo, 2017).

Imagine two distant stars A and B, a distance D apart in flat spacetime. Now suppose a spaceship starts moving away from A at time t_0, with a constant velocity $v < 1$ (recall that we are using natural units, in which $c = 1$). The spaceship comes to rest at a distance d from A. At this instant, the perturbation of spacetime appears, and we assume $R \ll d \ll D$. The perturbation pushes the spaceship at a constant acceleration a. Half-way between A and B, the distortion is modified, and the acceleration becomes $-a$. Finally, the spaceship comes to rest at a distance d from B, the perturbation disappears, and the spaceship moves at a constant speed v towards B.

In this scenario, let us calculate the proper (measured) time of the trip as seen by both an outside and an inside observer. An outside observer is an observer in flat spacetime, as shown by Equation 3. Therefore, we can calculate the expression for the observed time as we would in classical mechanics:

$$d = vt_1 \rightarrow t_1 = \frac{d}{v}$$

$$\frac{D - 2d}{2} = \frac{a{t_2}^2}{2} \rightarrow t_2 = \sqrt{\frac{D - 2d}{a}}$$

$$T = 2(t_1 + t_2) \rightarrow T = 2(\frac{d}{v} + \sqrt{\frac{D - 2d}{a}})$$

Now, for observers within the spaceship, it becomes a little more complicated. First consider how t_1 might change. During the first part of the trajectory, there is no perturbation, so the inside observer is also in flat spacetime. However, they are moving at a constant speed of v, which causes time dilation:

$$t_1 = \frac{d}{\gamma v}$$

This is simply a special relativity effect. Now, we need to consider what happens when the spacetime metric is no longer flat. For that, we introduce the notion of proper time within general relativity, which is simply defined as τ, where (Carroll, 2004)

$$d\tau^2 = -ds^2$$

Therefore, we must go back to the Alcubierre metric to understand what the proper time for the inside observer would be. For an observer within the spaceship, we have $z = z_0(t)$ and $dx = dy = 0$. Substituting these values in Equation 2, we obtain

$$ds^2 = -dt^2 + 0^2 + 0^2 + \left[dz_0 - \frac{dz_0}{dt} f(0) dt\right]^2 \rightarrow ds^2 = -dt^2$$

which implies

$$d\tau^2 = dt^2$$

In other words, the proper or measured time for an observer inside the warp bubble is simply the coordinate time: there is no time dilation. We can then write the total proper time measured by such an observer as

$$\tau = 2(\frac{d}{\gamma v} + \sqrt{\frac{D - 2d}{a}})$$

Finally, using the assumption of $R \ll d \ll D$, we obtain the approximation $\tau \approx T \approx 2\sqrt{D/a}$. This implies that the time as measured by both observers can be made arbitrarily small by increasing the value of a. Therefore, the spaceship may travel faster than light.

Stress-energy Required

Now that both the geometrical characteristics and the consequences of Alcubierre's metric have been explored and analyzed, it is only natural to ask what type of material is needed to create such an object. That is the main concern of this section.

Energy Conditions

The WEC (Weak Energy Condition) means, physically, that the local energy density is always positive. We can verify that it is violated by the warp drive (Lobo, 2017):

$$T_{\mu\nu} t^\mu t^\nu = -\frac{v^2}{32\pi}\left[\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2\right] < 0$$

The NEC (Null Energy Condition) for a null vector oriented along the $\pm\hat{z}$ directions takes the form

$$T_{\mu\nu} l^\mu l^\nu = -\frac{v^2}{8\pi}[\left(\frac{\partial f}{\partial x}\right)^2 + \left(\frac{\partial f}{\partial y}\right)^2] \pm \frac{v}{8\pi}(\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2})$$

which, averaged over the positive and negative directions, yields a negative value and therefore a violation of the NEC. Even if we do not average, the coefficient of the term linear in v must be nonzero somewhere in spacetime. At low velocities, this term will dominate, and the NEC will be violated in either the $+\hat{z}$ or the $-\hat{z}$ direction.

One may quantify the amount of energy condition violating matter in the warp bubble using the "volume integral quantifier" (Kar, Dadhich & Visser, 2004)

$$M_{warp} = \int \rho_{warp} d^3x = \int T_{\mu\nu} t^\mu t^\nu d^3x = -\frac{v^2}{12}\int (\frac{df}{dr})^2 r^2 dr$$

For the form function in Equation 4, we estimate,

$$M_{warp} \approx -v^2 R^2 \sigma$$

which shows that the negative energy required for the warp drive scales quadratically with the bubble's velocity and radius, and inversely with the bubble wall thickness.

Quantum Inequality

The quantum inequalities are energy constraints on the magnitude and extent of distributions of negative energy density. Their form comes from the uncertainty principle, which allows for negative energy to happen if it does not occupy too much space or exist for too much time (Ford, 1978; Ford & Roman, 1995).

When applied to the Alcubierre drive, the quantum inequalities show that, unless v is extremely large, the bubble wall thickness cannot be much above the Planck scale (where the Planck length is approximately equal to $1.616255 \times 10^{-35} m$). Additionally, it reveals that the energy required for a warp bubble is of the order of

$$E \leq -3 \times 10^{20} M_{galaxy} v$$

which is roughly ten orders of magnitude greater than the total mass of the entire visible universe (Pfenning & Ford, 1997) and clearly an absurd requirement.

Challenges and Problems

Here we briefly cover two of the biggest challenges regarding the Alcubierre drive, other than its energy requirements. Put differently, we suppose we can find the amount and type of material we need to build the warp bubble, and then study the problems still present in the solution.

The Horizon Problem

Not long after Alcubierre's discovery of the warp drive, it was shown that an observer on a spaceship cannot create nor control an Alcubierre bubble with $v > c$ around the ship (Krasnikov, 1998). The reason is that such an observer cannot causally affect or alter events outside of their future light cone, $|r| \leq t$, and, for the warp drive, any point on the outside front edge of the bubble is spacelike separated from the center (Lobo, 2017).

To understand why, we move to the proper reference frame of an observer at the center of the bubble. Using the transformation $z' = z - z_0(t)$, the metric is given by

$$ds^2 = -dt^2 + dx^2 + dy^2 + [dz' + (1 - f)vdt]^2$$

Consider a photon emitted along the $+Oz$ axis. Because it is a photon, $ds^2 = 0$. Due to its direction of travel, $dx^2 = dy^2 = 0$. This, together with the metric above, implies that $\frac{dz'}{dt} = 1 - (1 - f)v$. If the spaceship is at rest at the center of the bubble, then the photon initially has $\frac{dz'}{dt} = 1$, which, in natural units, is to be expected. However, at a point $z' = z_c'$ where $f = 1 - 1/v$, $\frac{dz'}{dt} = 0$. In other words, there is a point within the bubble in which the photon will stop moving relative to the bubble, and instead be carried along with it. A photon emitted from inside the spaceship will never reach the bubble's outside edge. Therefore, that edge lies outside the forward light cone of the spaceship. This points to the appearance of an event horizon.

This does not mean that the Alcubierre drive, if one could be created, could not be used for superluminal travel. It simply means that whatever actions required for the creation and control of the bubble must be taken beforehand by a different observer (one whose forward light cone contains the entire trajectory of the drive). It is worth noting that Krasnikov introduced a metric, the Krasnikov tube, to counter this difficulty. His solution allows the time for a round trip, as measured by clocks at the starting point, to be made arbitrarily short (Krasnikov, 1998).

Closed Timelike Curves

If we consider a spaceship traveling within a warp bubble of arbitrary velocity $v(t)$, the metric of the warp drive allows for superluminal travel, as we have seen previously. That raises the possibility of closed timelike curves (CTCs), which would imply the theoretical possibility of backwards time travel. Because of the causality paradoxes that come with such a possibility, this characteristic is considered a problem. Alcubierre's original solution does not possess CTCs, but Everett demonstrated their appearance through a simple modification of the original metric (Everett & Roman, 1997).

Recent Progress

This section is a summary of the advances made in warp drive research since the publication of Alcubierre's original article.

Five years after Alcubierre published his results, Van Den Broeck (1999) proposed a slight modification to his geometry. His goal was to make the energy requirements for a warp drive more reasonable. To that end, he tried to keep the surface area of the warp bubble microscopically small, while expanding the spatial volume inside of it. The new metric satisfies the quantum inequality, brings the same benefits as Alcubierre's drive, and only requires a total negative mass of the order of a few solar masses (together with a comparable amount of positive energy).

Shortly after, Natário (2002) showed that the need for expansion and contraction of volume elements in Alcubierre's spacetime is only a consequence of the choices he made, and not a general requirement for warp drive geometries. He introduced a warp drive in which the expansion (contraction) of the distances along the direction of motion is compensated by a contraction (expansion) of area elements in the perpendicular direction. Therefore, in the Natário warp drive, the volume elements are preserved. The essential property of the warp drive is revealed to be the change in distances, and not volumes, along the direction of motion.

Two years later, Lobo and Visser (2004) applied a linearized approach to the warp drive spacetime, which makes no assumptions about the nature of the bubble's operation (quantum or classical): the energy restrictions found are more generic than those previously analyzed. They found that the warp drive is an example of a "reaction-less drive", and that, for both the Alcubierre and the Natário drive, the energy condition violations persist even at arbitrarily low speeds. Finally, they discovered that, even in the weak-field limit, the warp drive is very tightly constrained (only conceivable at absurdly low velocities).

In his "Warp Field Mechanics 101", White (2011) recalculates Alcubierre's initial concept and argues that if the warp bubble were shaped like a torus, it would be much more energy efficient. He and his team at NASA are working with a warp field interferometer – a modified version of the Michelson-Morley interferometer – to demonstrate warp field phenomena. They are using lasers to perturb spacetime by one part in 10 million. Lee and Cleaver (2016) have argued that the interferometer is unable to spectrally resolve spacetime distortions.

McMonigal, Lewis, and O'Byrne (2012) examined the interactions between massive particles and the Alcubierre warp drive, as well as the effects of an accelerating or decelerating warp bubble. Particles with positive initial velocity with respect to the rest frame of the origin/destination of the ship obtain very high energy and experience very little proper time during their time in the bubble. When interacting with an accelerating bubble, a particle within the bubble will receive a velocity boost which increases/decreases the magnitude of their velocity if they are moving towards the front/rear of the bubble. A decelerating bubble has the opposite effect.

Recently, Lentz (2021) proposed that the conventional (non-exotic) energy sources could arrange the structure of spacetime in the form of a soliton, which would act like a warp bubble, contracting space in front of it and expanding it behind it. Following his discovery, Fell and Heisenberg (2021) designed their own positive-energy warp drive geometries. Earlier this year, Bobrick and Martire (2021) developed a model of a general warp drive spacetime in classical relativity. They presented a general model for subluminal, positive-energy, spherically symmetric warp drives, decreased the negative energy requirements of the Alcubierre drive by two orders of magnitude, and introduced a warp drive in which space capacity and the rate of time can be chosen in a controlled manner. Additionally, they showed that any warp drive is a shell moving inertially, implying any warp drive required propulsion, and that a specific class of subluminal, positive-energy, spherically symmetric warp drives can, in principle, be constructure based on the principles known to humanity today.

However, some theorists (Santiago, Schuster & Visser, 2021a) claim that such warp bubbles (those found by Lentz, Fell and Heisenberg, and Bobrick and Martire) have only been analyzed in the reference frame of Eulerian observers. Therefore, they are not guaranteed to only require positive energy as measured by any timelike observer (which is what the WEC requires). The same authors have recently used the warp drive as an "ansatz" for building a gravitationally induced stressor beam geometry (Santiago, Schuster & Visser, 2021b).

Conclusion

We began our exploration of warp drives with two purposes in mind: to evaluate their physical plausibility, and to understand the importance of research on the topic, regardless of plausibility.

Are Alcubierre drives physically plausible? Almost certainly not. There are two main reasons for this: they require exotic matter, and they require a very large amount of matter. Indeed, the high negative energy densities required do not seem even theoretically plausible: they would create the need for extremely thin bubble walls, which in turn require extreme amounts of energy. Even if the required amount of negative energy were available, there are no self-consistent warp drive solutions which can self-accelerate from zero velocities.

What about warp drives in general? As mentioned in Recent Progress, Bobrick and Martire recently found that both the need for negative energy density and the need for enormous amounts of energy are not a general property of warp drive spacetimes. Instead, such difficulties encountered in the Alcubierre drive, as well as in its modifications and in the Natário drive, come from choices made about the metrics, which can be avoided. This discovery creates hope that, although the Alcubierre drive seems unphysical, some subluminal, spherically symmetric warp drives might not be.

In short, it seems that the Alcubierre drive will never be technologically useful, but that different metrics for subluminal warp drives may be theoretically plausible. Regardless of that technological possibility, why should we care about warp drive research?

Warp drives, like wormholes, tractor beams and Krasnikov tubes, are good thought experiments, which force physicists to face the limits of their theories. In this case, warp drives drive us to the limits of general relativity and confront us with the definition of "superluminal travel", which can be more complicated than it seems. Additionally, it drives research on energy conditions and exotic materials, increasing interest on quantum field theory and quantum gravity (Hiscock, 2002). One suggested area for further investigation is that of warp drives in "beyond-GR" theories of gravity: can a reasonable theory of gravity allow superluminal warp drives to exist? If so, how should physicists face the causality violations implied? The warp bubbles found by Lentz, Fell and Heisenberg, and Bobrick and Martire should also be analyzed from the reference frame of a general observer (a non-Eulerian observer) to confirm the generality of their results, or lack thereof.

We conclude that warp drive research is relevant both due to its possible (even if unlikely) technological usefulness, and due to the interesting theoretical questions it raises for physicists working in a vast array of subjects.

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