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Fibonacci Sequence and Art: The Measure of Utilization During Art Movements in European History

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ABSTRACT

The goal of this study was to examine paintings from three different art movements--the Renaissance, Baroque, and Romantic eras--and determine if there were differences in the observed usage of the Fibonacci sequence during each period. By doing this, I hoped to further the understanding of the techniques and characteristics of paintings made during these art movements. Prior research shows that the Fibonacci sequence has been noticed in some major Renaissance pieces, such as the *Mona Lisa* and the *Vitruvian Man*; however, there is no apparent research done specifically on how this mathematical principle has appeared in other art movements, nor are there any existing comparisons made between art movements in regard to utilization of this sequence. Data was collected through content analysis based on random samples of paintings that I formulated. For each art movement, through content analysis, it was then determined whether or not each painting incorporated the Fibonacci sequence. The results had shown that the Renaissance era had utilized the sequence in 60% of its paintings, the Baroque in 40%, and the Romantic in 30%. Therefore, it can be concluded that the Fibonacci sequence had differed in how often it had been incorporated when comparing the Baroque and Romantic eras to the Renaissance; however, this is limited due to various factors during the analyzation process. Further research should try to examine various other art movements, and if modern era art is more prone to utilize the Fibonacci sequence as knowledge of this mathematical concept is more universally known.

Introduction

There are typical aspects that art historians look at when analyzing paintings, such as the year the piece was made, the art movement it belongs to, the lighting, potential symbolism, the meaning behind the work, etc. A factor that is not as commonly investigated, however, is how specific mathematical concepts are incorporated into art--one such idea being the Fibonacci sequence. The Fibonacci sequence is a mathematical tool that was discovered by Leonardo Fibonacci of Pisa, circa the year 1202 CE (Fisher & Rabung, 2009). The Fibonacci sequence is a list of numbers where each successive term is equivalent to the sum of the prior two numbers. Essentially, the sequence starts out 0, 1, 1, 2, 3, 5, 8, 13, etc., and continues infinitely in the same manner (Kemp, 2004). This sequence can be applied to art and photography in various ways, and numerous professionals discuss how the sequence is also often found naturally in science and the environment--such as in pine cones, nautilus shells, and sunflowers, to name a few. For example, with pine cones, their kernels align in two separate spirals that run in opposite directions. Its connection to the Fibonacci sequence is that in each grouping, the number of kernels is typically a number in the sequence, such as 8, 13, 21, etc. (Maor, 2020). Moving away from the Fibonacci sequence's presence in nature/photography, this research paper looks into the involvement of the Fibonacci sequence during three specific art movements--the Renaissance, Baroque, and Romantic--and seeing if the percent incorporation of the sequence via the application of the Fibonacci spiral differs between them.

Literature Review

Fibonacci Sequence

The Fibonacci sequence, other than simply possessing a specific pattern, also relates to several different mathematical principles, one such principle being the golden ratio. The golden ratio is approximately equivalent to the number 1.618, a number which the Fibonacci sequence infinitely approaches as one moves along the list of numbers. In essence, as you move down the successive terms in the sequence, the quotient of a term divided by the previous term in the list will get closer and closer to the golden ratio value of approximately 1.618 (Maor, 2020).

In addition to the concept of the golden ratio being involved with the Fibonacci sequence, there is also the involvement of spirals and the golden rectangle. The golden rectangle is a rectangle whose width and length are in the proportion of approximately 1.618 and is said to be "the most aesthetically pleasing of all rectangles" --which is thus why it frequently appears in art and architecture (Bell & Garofalo, 2003). Golden spirals are logarithmic spirals that can be approximated by the proportion of the golden ratio. Another spiral that follows the golden ratio, similarly to golden spirals, are Fibonacci spirals. Fibonacci spirals can be found via golden rectangles. A quarter circle is drawn through each box within the rectangle, each square having the side lengths of a number of the Fibonacci sequence. These quarter circles ultimately form the Fibonacci spiral (Jun-Sheng, 2019). These concepts take shape in naturally occurring places, such as in the crystallographic structure of DNA, in botanical phyllotaxis, in the curvature of elephant tusks, etc. (Boeyens & Thackeray, 2014). This paper, however, focuses on its incorporation in paintings, such as how it had been used in Leonardo da Vinci's, *The Mona Lisa* (Kemp, 2004).

Art Movements

Renaissance

The Renaissance era in art occurred in the 1300s until the 1500s in the continent of Europe. Characterized by the concepts of individualism, increasing one's awareness of nature, and a revival of classical learning, this time period is often referred to as a "rebirth" in values that are found in various areas of life--including art. The Renaissance era in Europe, although being home to a multitude of iconic artists and pieces, is most known for the work of three specific individuals in particular: Leonardo da Vinci (1452-1519), Michelangelo (1475-1564), and Raphael (1483-1520). The Renaissance had essentially ended with "The Sack of Rome" in 1527, but it is said that the influence of this movement had continued in Italy for the remainder of the sixteenth century ("Renaissance art," 2020).

Baroque

The Baroque era in art had occurred from around the 1600s until the 1700s in Europe, with different areas experiencing this movement at different times. Germany had encountered the effects of this movement in the later eighteenth century, as opposed to other areas that had done so earlier. The Baroque art movement is typically characterized by the desire of artists at the time to invoke emotion via appeals to the senses. Pieces made during this era prominently feature characteristics such as grandeur, drama, tension, etc. ("Baroque art and architecture," 2020).

Romantic

The Romantic era in art occurred from approximately the late 1700s to around the year 1850 in Europe. Within the context of the time period, Romanticism was typically seen as a reactionary art movement to the Enlightenment, and concepts such as "rationalism" and physical materialism. The Romantic art movement is characterized by artists' use

of emphasis on the ideas of the individual, irrationalism, imagination, etc. Compared to previous art movements, this period had been less rigid in certain artistic ideals, such as that of harmony and balance. The Romantic era focused on art being more imaginative, embracing nature, having its focal point being emotion rather than reason, etc. ("Romanticism," 2020).

Incorporation of Fibonacci Sequence in Art

Certain principles that go hand-in-hand with the Fibonacci sequence, such as the golden ratio and Fibonacci spirals, can be seen in art for centuries, dating back from ancient times until the modern era. In Mesopotamia, nearly 5,000 years ago, Sumerian architects had built their religious buildings, *ziggurats*, in the form of these spirals. In Greece, painters had decorated vases with an ornamental motif called "*guilloche*," which had adopted this same spiral shape. In the late nineteenth century, Vincent Van Gogh had even painted his famous *Starry Night* utilizing the spirals in colorful bursts in the sky. Artists have been using the Fibonacci sequence via these spirals, the number sequence itself, the golden ratio, etc., in their pieces throughout history. Their utilization seems to portray a product that is visually pleasing. This is likely due to the fact that the Fibonacci sequence is a natural concept that is commonly observed in the environment—it is not an inherently man-made concept (Cetcovik et al., 2012).

Gap, Research Question, and Hypothesis

As it is now, the amount of published, scholarly, and peer-reviewed work that exists on the Fibonacci sequence and art is rather limited. Thus, this lack of available information is where the "gap" that this paper focuses on had originated. The Fibonacci sequence being utilized in paintings during art movements other than the Renaissance has not been commonly written about, if at all. This factor, combined with a lack of comparisons regarding the usage of the Fibonacci sequence during different art movements, led to a curiosity on this subject matter. In pieces such as "The Measure of Beauty and the Beauty of Measure" by Elizabeth A. Fisher and John Rabung, and "Science in Culture" by Martin Kemp, the researchers only discuss how Leonardo da Vinci implemented the Fibonacci sequence in his pieces, *The Mona Lisa* and *The Vitruvian Man*. The lack of discussion on other art pieces, especially in art movements other than the Renaissance and what da Vinci had created, is what inspired the question that this paper seeks to suggest a possible answer to: "To what extent does the incorporation of the Fibonacci sequence in paintings differ between the Renaissance, Baroque, and Romantic art movements in European history?"

The expected answer to this question is that there is an apparent difference between the art movements, and that the Renaissance will incorporate the sequence the most frequently, and the Romantic era will utilize it the least. This is due to the fact that most research on this subject discusses how the sequence was used during the Renaissance, and no other art movements. In addition, the Romantic era had a major focus on imagination and moving away from artistic ideals, such as balance--thus why this time period utilizing this mathematical concept frequently is unlikely, as the Fibonacci sequence and spiral is a way of creating balance in art ("Romanticism," 2020).

Method

Approach and Design

To answer the goal of my project, which is to compare the percent usage of the appearance of the Fibonacci sequence in paintings during the Renaissance, Baroque, and Romantic art movements, I had decided that I would need to approach my research quantitatively, as I needed numerical values to compare. In order to do this, I determined that I would first have to take simple random samples in order to generalize the parent populations without bias, as it would not be feasible to look at every painting from each art movement. Thus, I utilized the Metropolitan Museum of Art's

website like a database in finding paintings. Using specific parameters in order to get paintings from each art movement--i.e., "Paintings," "Europe," "1400-1600 A.D.," and "European Paintings" in order to get Renaissance paintings--I first wrote down every piece, forming lists (see "Appendix A, B, and C"). With these lists for each movement, I numbered them in order of appearance on the website, and then used a random number generator in order to select ten distinct paintings from each of the three art movements (see "Appendix D"). As I was selecting the paintings, if a selected painting did not have a date that it was made, or that it was not made during the outlined time period, I omitted the painting. As I had specifically wanted to compare pieces from the Renaissance, Baroque, and Romantic, it was important that the paintings that I observed were from these art movements. The error that caused the need for omissions can be attributed to the fact that the Metropolitan Museum of Art's website does not have proper sections for each art movement, and that during the process of generating the populations of paintings, it might have provided paintings that have the proper characteristics for each era, but were not actually made during them. As a result, by taking these steps to ensure that I properly selected ten paintings from each art movement, I was able to obtain my samples of paintings with minimal selection bias.

Once I had gained my random samples of paintings for the Renaissance, Baroque, and Romantic eras, I then performed a content analysis on each piece. A content analysis is where a researcher will look at pre-existing sources, and inspect them to see if there are any recurring themes, etc. (Leedy & Ormond, 2015). For my project, I had focused specifically on trying to see if the selected paintings from each art movement had utilized the Fibonacci sequence via incorporation of the Fibonacci spiral, and so I found it fitting to perform content analyses. I had used reference photos of what this looked like on paintings that have already been scrutinized in order to ensure that I was analyzing the paintings that I had randomly selected properly. For each painting, I put a transparent Fibonacci spiral image on top of the piece (see "Appendix E") and oriented it until I was able to determine whether it was utilized or not. A major consideration during this process was making sure that the proportions of the Fibonacci spiral itself were not changed, as that would make the data collected inaccurate. I kept a thorough observation chart based on what I took note of regarding each painting. The dichotomous, or binary, variable that I kept track of here was whether or not the Fibonacci sequence was present. Essentially, a 0 or a 1 was assigned to each painting: 0 meant that the sequence was not incorporated, 1 meant that it was.

After I had analyzed each painting, and collected my observations, I had to calculate percentages in order to make generalizations about each art movement. Due to the total number of points that an art movement could receive is 10, based on the scaling of the variable being collected, I divided the number of points I got from each movement by 10. The resulting number was then converted into a percent. With these percentages, I was then able to make comparisons between the art movements. Whichever art movement had the highest percentage assigned to it had the highest observed incorporation of the Fibonacci sequence, and the opposite for the era assigned the smallest percentage. Keeping in mind the characteristics that had defined the Renaissance, Baroque, and Romantic art movements, I was then able to make conclusions that suggest an answer to my research question: "To what extent does the incorporation of the Fibonacci sequence in paintings during the Renaissance era differ from its usage in the Baroque and Romantic art movements in European history?"

Influential Studies

While gathering information to form a pool of background knowledge on my project's topic, I came across several studies that helped me to formulate--or at the very least, influence--my choice of method. One such piece was "The Real and the Ideal: An Investigation in Natural Pattern Design" by Gail Frederickson Bacon. In Bacon's graduate thesis, she elaborates on when analyzing art, there were certain laws/mathematical principles that naturally drew the eye. In her piece, she states, "I find that the images I am drawn to most include specific properties of ratio, symmetry, proportion and order…" (Bacon, 2012). This comment helped encourage my analysis process, as it helped me process what I observed in paintings better--those that included the Fibonacci sequence would most likely appear more pleasing to the eye. This had been an indicator that did not necessarily mean the painting incorporated the sequence, but I

had still thought it to be noteworthy. A study that had truly influenced my thought to use content analysis, however, was "The rise and fall of science education: a content analysis of science in elementary reading textbooks of the 19th century" by Peter Rillero. This piece is about researchers analyzing textbooks from American history in order to garner the percent/type of science taught in the United States in the past. In this study, the researchers had performed a content analysis in order to gather data to calculate percentages that would reflect the independent variable of the amount of scientific education in America in the past (Rillero, 2010). Although this paper's topic was not relevant to my research at all, it was still highly influential in me deciding to perform a content analysis as I similarly had wanted to focus on observing a theme in pre-existing materials. Ultimately, I was unable to find any studies that I deemed to be helpful in terms of analyzing paintings for the Fibonacci sequence; however, these aforementioned studies did help influence me in deciding what method would be best for what I was trying to achieve.

Results

After performing content analyses on each of the three art movements--the Renaissance, Baroque, and Romantic--I was able to see a clear trend and difference between them. During the Renaissance, it was perceived that six out of the ten paintings in the sample utilized the Fibonacci sequence. For the Baroque, it was observed that four out of the ten paintings had incorporated it, and for the Romantic it was three out of ten. As a result, it can then be calculated that the sample proportion of paintings that utilized the Fibonacci sequence was 60% for the Renaissance, 40% for the Baroque, and 30% for the Romantic. The results differ between these eras, with the greatest difference being between the Renaissance and Romantic eras. This suggests that there is a downward trend in incorporation from the Renaissance to the Romantic eras, chronologically. (See Table 1 below for the results collected, and "Appendix D" for the paintings that the numbers on the left-hand side correspond to).

Table 1: *Depiction of the Dichotomous Variable Measured for Each Art Movement*

Painting #	Renaissance	Baroque	Romantic
1	1	0	0
2	0	1	0
3	1	0	0
4	1	1	0
5	0	1	0
6	1	0	1
7	0	0	1
8	0	0	1
9	1	1	0
10	1	0	0

Note: 0 represents not incorporating the Fibonacci spiral; 1 represents incorporating the spiral

Discussion and Analysis

The results of the content analysis shows that there is an apparent downward trend in percent utilization of the Fibonacci sequence as one goes chronologically from art movement to art movement in regard to the three selected. These percentages calculated based on the samples can be used to make assumptions about the parent populations as the parent populations are all at least ten times greater than the sample size of ten. The results from the samples show that there is a 60% usage of the sequence during the Renaissance, 40% during the Baroque, and 30% during the Romantic era. As these three art movements illustrate a downward trend in utilization of the Fibonacci sequence, as demonstrated through fitting the Fibonacci spiral, as you approach closer to the modern era, it alludes to an interesting conclusion--time may have an effect on how this mathematical principle is used. This can be tied into what each of these three art movements had an emphasis on. The Renaissance, beginning in approximately the 1300s, is closer to when the sequence was first discovered, in 1202. As a result, it could have been more well-known during this time, and thus was incorporated the most during this era. The Romantic art movement, furthest from this date out of the three eras, is often characterized by its focus on imagination, nature, religion, etc., with many of the paintings being landscapes. Due to this, it is logical that it would utilize the Fibonacci sequence the least as math was not a focus in art during this time and would not commonly be associated with the characteristics that are typically matched with the Romantic period. Also, the prominence of the Fibonacci sequence in teaching/education might not have been as great during this time in comparison to the Renaissance.

Although the results from the method performed do go along with the expected results and the hypothesis of this research study, there are various limitations that may have either skewed or influenced the results. One such limitation is the fact that the population lists that the samples were selected solely from the Metropolitan Museum of Art. Two problems with this are that it is unlikely that they have a complete record of every painting from these art movements, and the fact that they did not have a clear way to select paintings from each art movement. As a result, it is possible that the paintings selected were not entirely representative of the art movements that it was said that they belong to. To overcome these limitations, it was determined that the population of paintings from this source was sufficient for the purpose of making generalizations about these art movements, and that the specific parameters that were plugged into their search engine were enough to as accurately as possible acquire paintings from each era. And so, although it is possible that these factors had an impact on the results, it is likely that they were insignificant.

Another limitation had been the size of the samples that were observed during the content analysis. Ten paintings had been looked at from each art movement, which is relatively small. However, since the population size of paintings for the Baroque period had been one hundred, it was necessary to pick a sample size of ten so that the populations would all be at least ten times greater than the samples so that generalizations could be made properly about the populations.

Conclusion

From the data collected, considering the limitations of the research, it can be suggested that the Renaissance, Baroque, and Romantic eras differ in percent utilization of the Fibonacci sequence, the greatest difference being between the Renaissance and Baroque eras. In addition to the fact that it was able to be determined that these art movements did differ in the frequency of how often the Fibonacci sequence was incorporated--made evident from aspects of the paintings that either do or do not fit the application of the Fibonacci spiral--it was noted that there was a chronological downward progression in how frequently this mathematical principle was observed. From the samples, it was calculated that the Renaissance had incorporated the Fibonacci sequence the most, then the Baroque, and the Romantic era the least. Ultimately, the data provided from the research method performed was able to adequately answer the question that this study was rooted in as it showed that there does appear to be a difference in utilization of the Fibonacci sequence between the art movements, no two eras incorporating it the same amount.

Future Directions

Regarding future directions in this topic of study, there is much more that can be looked into. The Fibonacci sequence was only formally discovered in the year 1202, and of course there has now been the introduction of computers and modern technology that has had a major impact on how some art is made--i.e., graphic and computer-generated art. As noted by the research in this study, there appears to be a downward trend in how frequently the Fibonacci is observed in paintings dating from the 1300s to the year 1850. So, it would be interesting to look at paintings made both before the formal discovery of this mathematical principle, and paintings that were made after the advent of modern technology and computers. By looking at paintings made before the year 1202, if it is noticed that the Fibonacci sequence was used, it could mean that the incorporation of this mathematical principle is mostly coincidental, and in fact appear solely by chance rather than artists actually knowing of its existence. On the other hand, by looking at modern art that was made either via computers or by computer-generated programs, it would be interesting to see if there would be a spike in its incorporation in art due to technology having a basis in math--and the Fibonacci sequence, in its entirety, is based on math.

It would, in addition, be intriguing to consider how location could influence how the Fibonacci sequence was incorporated. As this sequence was discovered by an Italian man--Leonardo Pisano--this could be why the sequence is commonly found utilized in paintings from European countries. It would be interesting to see if the sequence is far less in paintings from other continents, or if, once again, the appearance of this mathematical principle is purely coincidental. It is a possibility that the Fibonacci spiral might be more apparent in other areas after overseas travel became more frequent; it is also possible that there was no effect, as it is still not necessarily certain whether the usage of the Fibonacci sequence in paintings is purposeful or not.

Another route that future researchers could potentially take is to create a program that is able to more accurately analyze paintings to see if the Fibonacci spiral could be properly applied to them. Although it is a partially subjective process--the researcher must deem what are major aspects of the painting and determine whether or not it fits the Fibonacci spiral well enough--it would be interesting to see if a computer program could be made to eliminate some of the bias in the analyzation process.

Implications

Through this research study, and the conclusions that were able to be drawn, the significance is that this could potentially have an impact on the current understanding of the Renaissance, Baroque, and Romantic if further research was to be conducted. Although the connection between math and art could be purely coincidental, it must be considered that maybe it is not. It is entirely possible that the Fibonacci sequence observed in these paintings might have been included on purpose, and not completely by chance, based on the relevance of mathematical principles during the time and area. In addition, this study provides potential evidence of another way that these art movements differ, which lends to a deeper understanding of these past eras. By learning more about specifics of the past, specifically art, you can learn a lot about the culture. Art is often reflective of the time—as seen by the Romantic era being a movement in reaction to “Rationalism”—and so by learning about what was prominent in art in the past, one can get a better understanding of the time. The findings of the research conducted, although do not definitively prove anything, suggest a new field of focus in art during the Renaissance, Baroque, and Romantic eras that should be further investigated.

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