

# Determining the Relationship Between the Velocity and Drag Coefficient of a Model Rocket

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## ABSTRACT

There has been a significant uptick in interest among the wider public toward space and its associated technologies. Despite this, there is still a significant lack of public resources discussing the more nuanced areas of rocketry. One such area is the behavior of air around a rocket as its speed increases. The changing speed causes the air around the rocket to flow differently, resulting in different drag characteristics. This paper studies this relationship. In keeping with the focus on accessibility, this paper will use a model rocket instead of the full-size version. This paper finds that there is a negative correlation between speed and air drag on a rocket.

## Nomenclature

$A$	=	reference area	$\text{m}^2$
$a$	=	total acceleration of object	$\text{m s}^{-2}$
$c_d$	=	drag coefficient	
$F_d$	=	drag force	N
$g$	=	Earth-surface gravitational acceleration	$-9.80665 \text{ m s}^{-2}$
$h$	=	height above sea level	m
$h_a$	=	height above launch point	m
$h_i$	=	height of initial launch point above sea level	m
$M_d$	=	Molar mass of dry air	$0.0289652 \text{ kg/mol}$
$m$	=	mass of object	kg
$p$	=	absolute pressure	Pa
$p_0$	=	air pressure at sea level	Pa
$R$	=	universal gas constant	$8.31446 \text{ J K}^{-1} \text{ mol}^{-1}$
$T$	=	temperature	K
$u$	=	flow speed of object relative to fluid	m/s
$\rho$	=	mass density of air	$\text{kg m}^{-3}$

## Introduction

Recent innovations in spaceflight as well as an increase in space publicity have stoked great interest among the broader public in space and its related technologies. The most prominent of these technologies are rockets, as they are central for the delivery of satellites and other space payloads. During flight, they are subject to a myriad of aerodynamic forces.

However, there is a severe lack of accessible resources for the public pertaining to the aerodynamics of rockets during flight. This is due to a variety of factors, such as the cost-prohibitive nature of the field, laws such as ITAR, and

company secrecy. Model rockets are by far the best avenue for the public to experience the science of rocketry. They are subject to most of the same aerodynamic forces as full-size launch vehicles while being much more affordable for testing. Unfortunately, I have also found a lack of studies around the aerodynamics of model rockets, especially the behavior of air drag on a model rocket during flight. This study seeks to remedy that. The force of drag through air can be described using the following equation:

Equation 1:

$$F_d = \frac{1}{2} \rho u^2 A c_d$$

In the above equation, the drag coefficient  $c_d$  quantifies the drag of an object in air. A lower  $c_d$  indicates an object will have less drag.

It is difficult or impossible to calculate the  $c_d$  for model rockets without experimental data. Moreover, the  $c_d$  is also not constant during flight and varies with the velocity of the rocket (Niskanen, 2013).

This experiment aims to determine the relationship between velocity and the drag coefficient of a model rocket. This relationship will be used to help predict the flight behavior of model rockets. These findings will also provide insight for the general public into the aerodynamic forces experienced by full-size rockets.

## Design and Methodology

### Hypothesis

As the velocity of the rocket increases, the drag coefficient will increase exponentially.

### Testing Apparatus

#### Rocket

A diagram of the rocket and a catalog of its parts with weight and material information can be seen in Figure 1 and Figure 2.

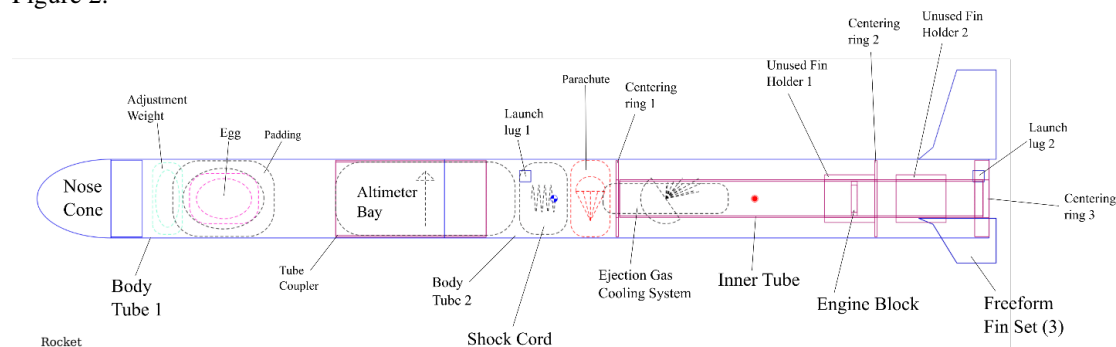
























Figure 1. Cutaway of the Rocket

	Nose cone	Styrofoam (generic EPS) (0.02 g/cm <sup>3</sup> )	Ellipsoid	Len: 6.2 cm	Mass: 20 g
	Body tube 1	Cardboard (0.68 g/cm <sup>3</sup> )	Dia <sub>in</sub> 6.4 cm Dia <sub>out</sub> 6.6 cm	Len: 28 cm	Mass: 29.4 g
	Adjustment Weight		Dia <sub>out</sub> 6 cm		Mass: 0 g
	Egg		Dia <sub>out</sub> 4.2 cm		Mass: 61 g
	Padding		Dia <sub>out</sub> 6.35 cm		Mass: 19.2 g
	Tube coupler	Cardboard (0.68 g/cm <sup>3</sup> )	Dia <sub>in</sub> 6.15 cm Dia <sub>out</sub> 6.45 cm	Len: 12.6 cm	Mass: 29.1 g
	Altimeter Bay		Dia <sub>out</sub> 6.14 cm		Mass: 71.8 g
	Body tube 2	Cardboard (0.68 g/cm <sup>3</sup> )	Dia <sub>in</sub> 6.4 cm Dia <sub>out</sub> 6.6 cm	Len: 45.7 cm	Mass: 50.7 g
	Launch lug 1	Polycarbonate (Lexan) (1.2 g/cm <sup>3</sup> )	Dia <sub>in</sub> 0 cm Dia <sub>out</sub> 0.95 cm	Len: 0.95 cm	Mass: 2.8 g
	Launch lug 2	Polycarbonate (Lexan) (1.2 g/cm <sup>3</sup> )	Dia <sub>in</sub> 0 cm Dia <sub>out</sub> 0.95 cm	Len: 0.95 cm	Mass: 1.4 g
	Freeform fin set (3)	Polycarbonate (Lexan) (1.2 g/cm <sup>3</sup> )	Thick: 0.312 cm		Mass: 55.7 g
	Parachute	Ripstop nylon (0.27 g/m <sup>2</sup> )	Dia <sub>out</sub> 59 cm	Len: 3.3 cm	Mass: 28 g
	Shroud Lines	Elastic cord (round 2 mm, 1/16 in) (1.8 g/m)	Lines: 8	Len: 62 cm	
	Shock cord	Elastic cord (round 2 mm, 1/16 in) (1.8 g/m)		Len: 466 cm	Mass: 24.9 g
	Inner Tube	Cardboard (0.68 g/cm <sup>3</sup> )	Dia <sub>in</sub> 2.9 cm Dia <sub>out</sub> 3.16 cm	Len: 30.5 cm	Mass: 25.7 g
	Engine block	Cardboard (0.68 g/cm <sup>3</sup> )	Dia <sub>in</sub> 2.3 cm Dia <sub>out</sub> 2.9 cm	Len: 0.5 cm	Mass: 0.8 g
	Ejection Gas Cooling System		Dia <sub>out</sub> 2.5 cm		Mass: 16 g
	Centering ring 1	Plywood (birch) (0.63 g/cm <sup>3</sup> )	Dia <sub>in</sub> 3.15 cm Dia <sub>out</sub> 6.4 cm	Len: 0.2 cm	Mass: 3.07 g
	Unused Fin Holder 1	Polystyrene (1.05 g/cm <sup>3</sup> )	Dia <sub>in</sub> 3.16 cm Dia <sub>out</sub> 4 cm	Len: 4.2 cm	Mass: 11 g
	Unused Fin Holder 2	Polycarbonate (Lexan) (1.2 g/cm <sup>3</sup> )	Dia <sub>in</sub> 3.16 cm Dia <sub>out</sub> 4 cm	Len: 4.2 cm	Mass: 11 g
	Centering ring 2	Plywood (birch) (0.63 g/cm <sup>3</sup> )	Dia <sub>in</sub> 3.15 cm Dia <sub>out</sub> 6.4 cm	Len: 0.2 cm	Mass: 3.07 g
	Centering ring 3	Plywood (birch) (0.63 g/cm <sup>3</sup> )	Dia <sub>in</sub> 3.15 cm Dia <sub>out</sub> 6.4 cm	Len: 1.2 cm	Mass: 18.6 g

### *Rocket Motor*

An F50-6T rocket motor from Aerotech is used for Launch 1. An F52-5T rocket motor from Aerotech is used for Launch 2.

### *Rocket Flight*

A diagram of the model rocket's flight is shown in Figure 3. This investigation will only use data from the rocket's coasting phase.

### *Launchpad Setup*

A labeled picture of the rocket on the launchpad can be seen in Figure 4.

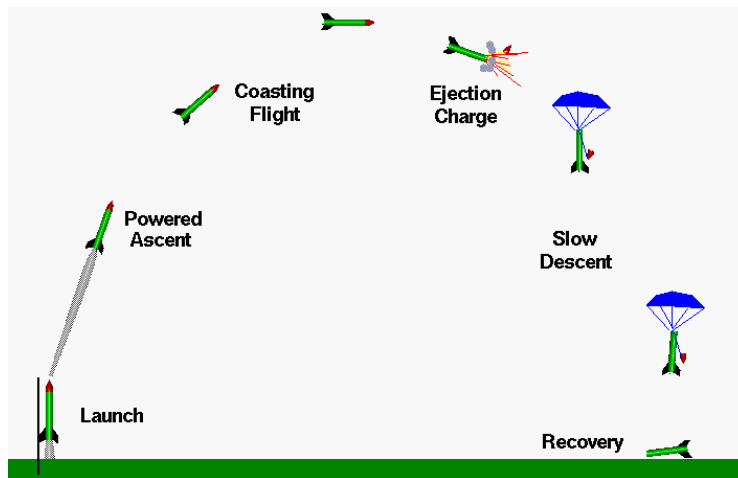


Figure 3. Diagram of Rocket Flight. This diagram is sourced from (NASA, n.d.).



### Measurement Devices

A *Pnut* altimeter from Perfectflite is used to record the vertical height of the rocket during launch. The *Pnut* is placed inside the altimeter bay shown in Figure 1. The *Pnut* records  $h_a$  every 0.05 seconds based on the change in the surrounding air pressure. It has an altitude measurement accuracy of  $\pm(0.1\% \text{ of reading} + 1 \text{ foot})$ .

A digital scale is used to record the mass  $m$  of the rocket. It has a measurement accuracy of  $\pm 0.00001 \text{ kg}$ .

The rocket was modeled in *OpenRocket* (Niskanen et al., 2012/2021) to measure values related to the rocket's surface area.

## Methods

1. The model rocket was constructed as shown in Figure 1.
2. The altimeter bay is loaded with the *Pnut* altimeter and slid into the tube coupler.
3. The shock cord and parachute are packed into the top of body tube 2. Body tube 1 and 2 are then joined together by the tube coupler.
4. An F50-6T rocket motor from Aerotech is inserted into the inner tube until it butts up against the engine block.
5. The current launch conditions are recorded from Flowx (Enzure Digital Weather App, 2021) and AccuWeather (AccuWeather, 2021).
6. An igniter is inserted into the rocket motor. The assembled rocket is then taken to the launchpad, where it is slid onto the launch rod.
7. The igniter is connected to electrical contacts. The *Pnut* altimeter is switched on.
8. After walking a safe distance away from the rocket, I send electric current via the electrical contacts through the igniter, igniting the rocket motor and beginning the launch.
9. The rocket motor will burn, launching the rocket. After the motor burns out the rocket will coast, after which an ejection charge in the motor will fire the parachute out of the rocket.
10. After the rocket drifts to the ground, I retrieve it and weigh it to get the  $m$  values for each launch.
11. Export the data from the *Pnut* altimeter to my computer, after which the altimeter is switched off.
12. The used motor is removed from the rocket.
13. Steps 2-11 are repeated with an F52-5T rocket motor from Aerotech.

## Data Collection and Processing

### Raw Data

Air pressure  $p_0$  and temperature  $T$  are given by Flowx (Enzure Digital Weather App, 2021) and AccuWeather (AccuWeather, 2021) in hPa and  $^{\circ}\text{C}$  respectively. The collected data is written in column 2 and 4 of Table 1 and Table 2.

The initial height  $h_i$  of the launch sight was recorded from Google Earth (Google, n.d.) in meters. The collected value is given in column 6 of Table 1 and Table 2.

To get the reference area  $A$  in  $\text{m}^2$  the rocket was modeled in OpenRocket as shown in Figure 1. The value for  $A$  was then exported from OpenRocket. The collected value is given in column 7 of Table 1 and Table 2.

To get the mass  $m$  in kg of the rocket after motor burnout, I first modeled the rocket in OpenRocket as shown in Figure 1. In OpenRocket, I selected the motor used for each launch. The value for  $m$  after motor burnout was then exported from OpenRocket. The collected values are given in column 8 of Table 1 and Table 2.

Table 1. Launch 1 Conditions

Date [m/d/y]	$p_0$ [hPa]	$p_0$ [Pa]	$T$ [ $^{\circ}\text{C}$ ]	$T$ [K]	$h_i$ [m]	$A$ [ $\text{m}^2$ ]	$m$ [kg]	Motor Flown
3/7/2021	1027.4	102740	9	282.15	25	0.003421	0.53941	F50-6T

Table 2. Launch 2 Conditions

Date [m/d/y]	$p_0$ [hPa]	$p_0$ [Pa]	$T$ [ $^{\circ}\text{C}$ ]	$T$ [K]	$h_i$ [m]	$A$ [ $\text{m}^2$ ]	$m$ [kg]	Motor Flown
3/21/2021	1033	103300	11	284.15	25	0.003421	0.56801	F52-5T

The *Pnut* altimeter records  $h_a$  in feet every 0.05 seconds. This investigation will restrict the range of raw data to the rocket's coast phase. A sample of the restricted range of raw data is given below in Table 3 and Table 4. The full restricted range is given in the Appendix.

Table 3. Sample of Time and  $h_a$  Data from Launch 1

Time [s]	$h_a$ [ft]
4.9	899
4.95	905
5	911
5.05	917
5.1	923
5.15	928
5.2	935
5.25	941
5.3	947
5.35	952
5.4	958
5.45	963
5.5	968
5.55	974
5.6	979
5.65	984
5.7	988
5.75	993
5.8	998
5.85	1004
5.9	1007

Table 4. Sample of Time and  $h_a$  Data from Launch 2

Time [s]	$h_a$ [ft]
3.5	642
3.55	654
3.6	660
3.65	671
3.7	680
3.75	690
3.8	700
3.85	707
3.9	715
3.95	724
4	731

4.05	739
4.1	748
4.15	758
4.2	768
4.25	775
4.3	783
4.35	791
4.4	801
4.45	809
4.5	817

### Assumptions

This investigation will assume that the horizontal distance traveled by the rocket was negligible, as most of its travel was vertical. This investigation will also assume that there was no wind. By accepting these assumptions, the vertical velocity of the rocket calculated from  $h_a$  can be used as  $u$ , the flow speed of the rocket relative to the atmosphere.

Humidity influences air density, which helps to determine the rocket's drag coefficient. According to Niskanen (Niskanen, 2013), the effect of humidity on air density is negligible, as the difference in air density between dry air and saturated air at standard conditions is less than 1%. Therefore, this study will ignore the effects of humidity on the drag coefficient.

This study will assume that the behavior of the  $c_d$  of the rocket will be the same regardless of whether the rocket motor is burning during flight. By accepting this assumption, the used data can be restricted to the rocket's coast phase, simplifying calculations.

### Data Processing

All calculations during data processing were made using Microsoft Excel. The values used were the exact values and not the rounded values given in the tables. The values were only rounded at the end apart from the % uncertainty.

Table 5. Sample of Processed Data from Launch 1

Time [s]	$h_a$ [m]	$h$ [m]	smoothed $h$ [m]	$u$ [m/s]	$a$ [ $m\ s^{-2}$ ]	$p$ [Pa]	$\rho$ [ $kg\ m^{-3}$ ]	$c_d$
4.9	274.0152	299.0152	298.9130	39.8269	-11.9344	99088.01	1.223434	0.345754229
4.95	275.844	300.844	300.8895	39.2302	-11.9344	99064.3	1.223141	0.356437913
5	277.6728	302.6728	302.8360	38.6335	-11.9344	99040.96	1.222853	0.367620483
5.05	279.5016	304.5016	304.7528	38.0367	-11.9344	99017.98	1.222569	0.379333490
5.1	281.3304	306.3304	306.6397	37.4400	-11.9344	98995.36	1.22229	0.391611019
5.15	282.8544	307.8544	308.4968	36.8433	-11.9344	98973.1	1.222015	0.404489935
5.2	284.988	309.988	310.3240	36.2466	-11.9344	98951.2	1.221745	0.418010165
5.25	286.8168	311.8168	312.1215	35.6498	-11.9344	98929.67	1.221479	0.432215005
5.3	288.6456	313.6456	313.8890	35.0531	-11.9344	98908.5	1.221218	0.447151467
5.35	290.1696	315.1696	315.6268	34.4564	-11.9344	98887.69	1.220961	0.462870667
5.4	291.9984	316.9984	317.3347	33.8597	-11.9344	98867.24	1.220708	0.479428267
5.45	293.5224	318.5224	319.0127	33.2630	-11.9344	98847.16	1.22046	0.496884965
5.5	295.0464	320.0464	320.6610	32.6662	-11.9344	98827.43	1.220217	0.515307054

5.55	296.8752	321.8752	322.2794	32.0695	-11.9344	98808.07	1.219978	0.534767051
5.6	298.3992	323.3992	323.8679	31.4728	-11.9344	98789.06	1.219743	0.555344411
5.65	299.9232	324.9232	325.4266	30.8761	-11.9344	98770.42	1.219513	0.577126342
5.7	301.1424	326.1424	326.9555	30.2793	-11.9344	98752.14	1.219287	0.600208728
5.75	302.6664	327.6664	328.4546	29.6826	-11.9344	98734.21	1.219066	0.624697188
5.8	304.1904	329.1904	329.9238	29.0859	-11.9344	98716.65	1.218849	0.650708284
5.85	306.0192	331.0192	331.3632	28.4892	-11.9344	98699.45	1.218636	0.678370914
5.9	306.9336	331.9336	332.7727	27.8924	-11.9344	98682.61	1.218428	0.707827908

Table 6. Sample of Processed Data from Launch 2

Time [s]	$h_a$ [m]	$h$ [m]	smoothed $h$ [m]	$u$ [m/s]	$a$ [ $m\ s^{-2}$ ]	$p$ [Pa]	$\rho$ [ $kg\ m^{-3}$ ]	$c_d$
3.5	195.6816	220.6816	220.1078	60.1459	-13.2566	100602.2	1.233386	0.2567521415
3.55	199.3392	224.3392	223.0985	59.4831	-13.2566	100566	1.232943	0.2626005129
3.6	201.168	226.168	226.0561	58.8202	-13.2566	100530.2	1.232505	0.2686477422
3.65	204.5208	229.5208	228.9805	58.1574	-13.2566	100494.9	1.232071	0.2749029464
3.7	207.264	232.264	231.8718	57.4946	-13.2566	100460	1.231643	0.2813757707
3.75	210.312	235.312	234.73	56.8317	-13.2566	100425.5	1.23122	0.2880764261
3.8	213.36	238.36	237.555	56.1689	-13.2566	100391.4	1.230802	0.2950157299
3.85	215.4936	240.4936	240.3469	55.5061	-13.2566	100357.7	1.230389	0.3022051487
3.9	217.932	242.932	243.1056	54.8433	-13.2566	100324.4	1.229981	0.3096568464
3.95	220.6752	245.6752	245.8312	54.1804	-13.2566	100291.5	1.229578	0.3173837355
4	222.8088	247.8088	248.5237	53.5176	-13.2566	100259.1	1.22918	0.3253995331
4.05	225.2472	250.2472	251.183	52.8548	-13.2566	100227	1.228787	0.3337188215
4.1	227.9904	252.9904	253.8091	52.1919	-13.2566	100195.4	1.228399	0.3423571150
4.15	231.0384	256.0384	256.4022	51.5291	-13.2566	100164.1	1.228016	0.3513309322
4.2	234.0864	259.0864	258.962	50.8663	-13.2566	100133.3	1.227638	0.3606578747
4.25	236.22	261.22	261.4888	50.2034	-13.2566	100102.9	1.227265	0.3703567139
4.3	238.6584	263.6584	263.9824	49.5406	-13.2566	100072.9	1.226897	0.3804474851
4.35	241.0968	266.0968	266.4428	48.8778	-13.2566	100043.3	1.226535	0.3909515913
4.4	244.1448	269.1448	268.8702	48.2149	-13.2566	100014.1	1.226177	0.4018919169
4.45	246.5832	271.5832	271.2643	47.5521	-13.2566	99985.31	1.225824	0.4132929521
4.5	249.0216	274.0216	273.6254	46.8893	-13.2566	99956.93	1.225476	0.4251809304

### Processing Altimeter and Launch Condition Data

The original height data given by the *Pnut* altimeter were in feet. These were converted to meters. The altimeter records the rocket's height above launch point ( $h_a$ ). This must be converted to height above sea level  $h$  using the equation:

Equation 2:

$$h = h_i + h_a$$

A sample of the resulting values of  $h$  after processing are given in column 3 of Table 5 and Table 6. The full data is given in the Appendix.



$p_0$  and  $T$  data given by Flowx (Enzure Digital Weather App, 2021) and AccuWeather (AccuWeather, 2021) are in hPa and  $^{\circ}\text{C}$  respectively. These were converted to Pa and K. The resulting values are given in column 3 and 5 of Table 1 and Table 2.

### Smoothing Height Data

The height data produced by the *Pnut* altimeter is noisy. The noise must be smoothed out to prevent it from affecting the rest of the calculations. The flight of a rocket during its coasting phase can be approximated with a quadratic equation. In Excel, I graphed Time on the X axis and  $h$  on the Y axis. I then made a quadratic line of best fit. The resulting graph and best-fit line are in Figure 5 for Launch 1 and Figure 6 for Launch 2. To get the smoothed  $h$  values, I plugged in Time for  $x$  in the best fit equations in Figure 5 and Figure 6. A sample of the smoothed  $h$  values are given in column 4 of Table 5 and Table 6.

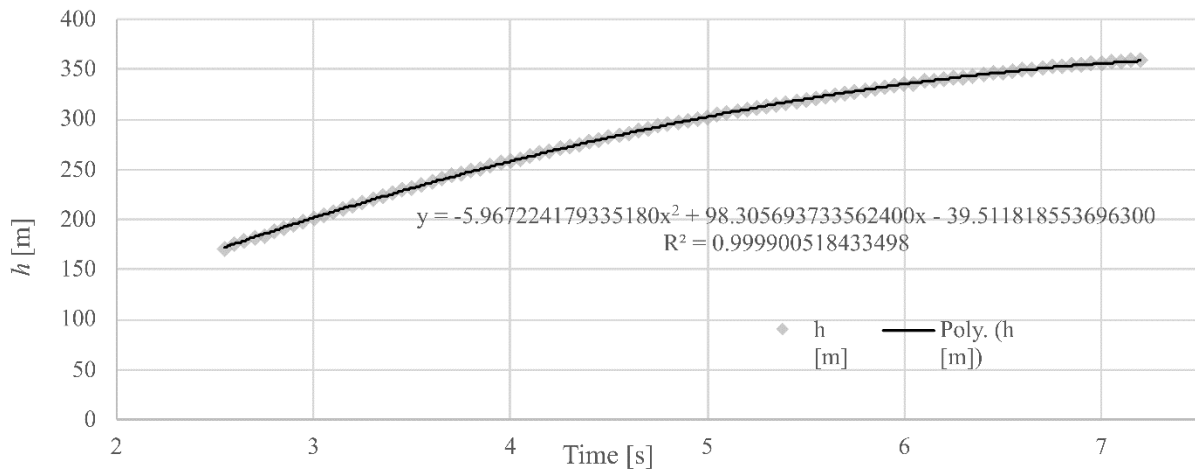


Figure 5. Altitude of Rocket as a Function of Time During Coasting Phase of Launch 1

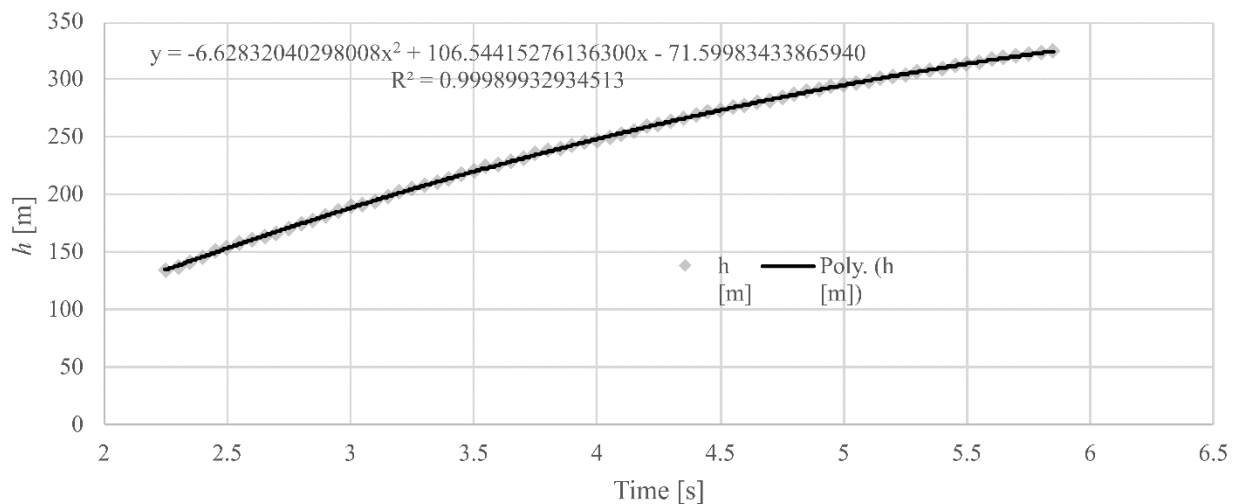


Figure 6. Altitude of Rocket as a Function of Time During Coasting Phase of Launch 2

### Calculating Velocity $u$ and Acceleration $a$

To approximate a measurement of the rocket's instantaneous velocity, I found the difference between the smoothed  $h$  value immediately before and after each row, then divided the result by difference in Time. A sample of the resulting values of  $u$  are given in column 5 of Table 5 and Table 6.

To approximate a measurement of the rocket's instantaneous acceleration, I found the difference between the  $u$  value immediately before and after each row, then divided the result by difference in Time. The resulting values of  $a$  are given in column 6 of Table 5 and Table 6.

### Calculating Drag Coefficient $c_d$

The total forces acting on the rocket after motor burnout are represented by the following equation:

**Equation 3:**

$$ma = F_d + mg$$

By rearranging Eq. (3) the force of drag  $F_d$  can be calculated with the following equation:

**Equation 4:**

$$F_d = m(a - g)$$

Substituting Eq. (1) for  $F_d$  in Eq. (4) and rearranging to solve for  $c_d$  results in the following equation adapted from (Milligan, 2012):

**Equation 5:**

$$c_d = \left| \frac{2m(a-g)}{\rho u^2 A} \right|$$

The value for  $\rho$ , the mass density of air, can be determined using the following equation:

**Equation 6:**

$$\rho = \frac{pM_d}{RT}$$

$p$  can be found using the following equation:

**Equation 7:**

$$p = p_0 e^{\left(\frac{ghM_d}{TR}\right)}$$

Solving for each of these values and substituting them into Eq. (5) results in the  $c_d$  values for each row. The calculated values for  $p$ ,  $\rho$ , and  $c_d$  are given in columns 7, 8, and 9 respectively of Table 5 and Table 6.

### Uncertainties

The uncertainties for  $p_0$  [Pa],  $T$  [K],  $h_i$  [m],  $A$  [m<sup>2</sup>], and  $m$  [kg] will be assumed to have an uncertainty of +/- their last significant figure before any conversion or the accuracy of their measurement device. They are shown in Table 7.

**Table 7.** Launch Conditions Uncertainties

$p_0$ [Pa]	$T$ [K]	$h_i$ [m]	$A$ [m <sup>2</sup> ]	$m$ [kg]
±10	±1	±1	±0.000001	±0.00001

A simplified method to find the uncertainty for  $h_a$  in meters is by putting the highest value of  $h_a$  recorded through the following equation based on the accuracy of the *Pnut* altimeter:

**Equation 8:**

$$\pm(h_a \times 0.1\% + 0.3048)$$

To find the uncertainty for  $h$ , add the uncertainties of  $h_a$  and  $h_i$ . To find the uncertainty for  $u$ , double the uncertainty of  $h_a$ . To find the uncertainty for  $a$ , multiply the uncertainty of  $h_a$  by 4.

To find the uncertainties for  $p$ ,  $\rho$ , and  $c_d$ , I input their respective equations into an Uncertainty Calculator (Truong, 2021) and filled in the largest recorded values for each variable along with their uncertainties. The resulting uncertainties for each launch rounded to 1 significant figure are recorded in Table 8 and Table 9.

**Table 8.** Launch 1 Data Uncertainties

$h_a$ [m]	$h$ [m]	$u$ [m/s]	$a$ [m s <sup>-2</sup> ]	$p$ [Pa]	$\rho$ [kg m <sup>-3</sup> ]	$c_d$
±1	±2	±3	±6	±34	±4	±0.5

**Table 9.** Launch 2 Data Uncertainties

$h_a$ [m]	$h$ [m]	$u$ [m/s]	$a$ [m s <sup>-2</sup> ]	$p$ [Pa]	$\rho$ [kg m <sup>-3</sup> ]	$c_d$
±1	±2	±3	±5	±32	±3	±0.4

Analysis

$u$  can be plotted against  $\ln(c_d)$ , as shown in Figure 7 and Figure 8.

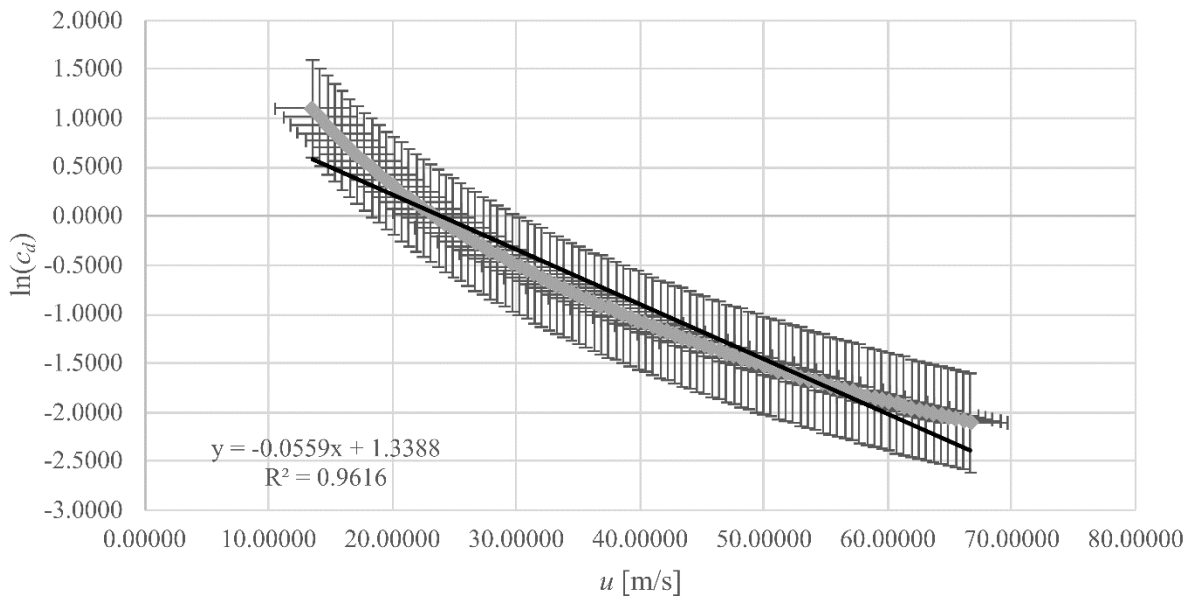


Figure 7.  $\ln(c_d)$  As a Function of  $u$  for Launch 1

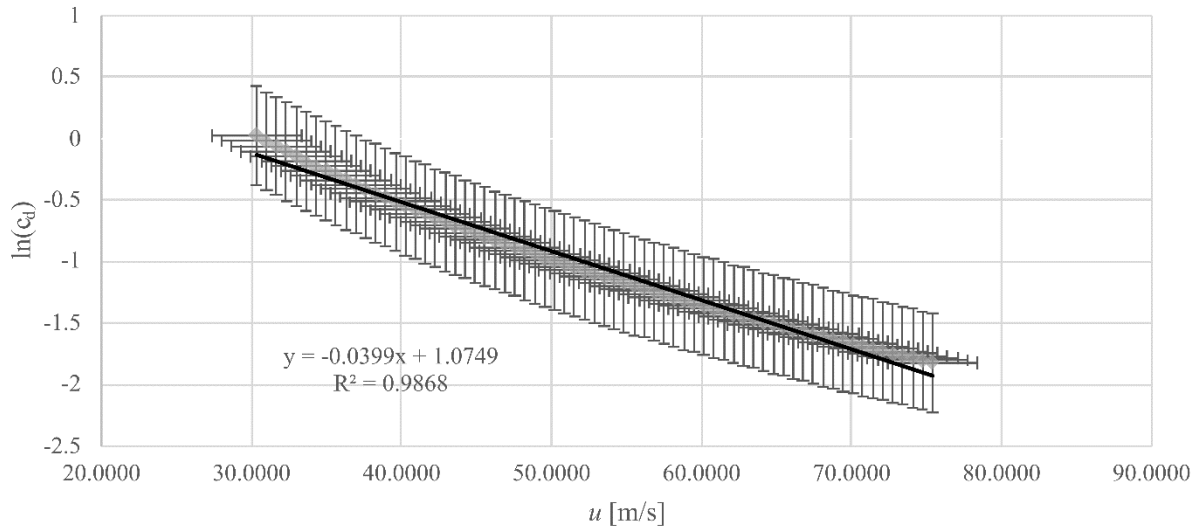


Figure 8.  $\ln(c_d)$  As a Function of  $u$  for Launch 2

## Conclusion

The data is not very precise, as the calculated errors are quite large compared to the calculated values. After the data was processed, it can be concluded that the results only support the hypothesis to a partial extent. Table 5 and Table 6 already show that as velocity  $u$  decreases, the drag coefficient  $c_d$  increases, suggesting a negative relationship. In order to have a linear correlation, the natural log of  $c_d$  was plotted against  $u$  in Figure 7 and Figure 8. The lines of best fit for these graphs indicate a linear relationship between  $u$  and  $\ln(c_d)$ . The graphs also indicate that there is an exponential relationship between  $u$  and  $c_d$ , since log plots linearize exponential relationships. This supports the hypothesis. Nevertheless, the hypothesis also predicted that there would be a positive relationship between  $u$  and  $c_d$ . Instead, the data shows a clear negative relationship between  $u$  and  $\log(c_d)$ , and therefore shows a negative exponential relationship between  $u$  and  $c_d$ .

The results from this study suggest that a model rocket performs best at high velocities. It can be theorized that faster speeds cause the air around the rocket to flow smoother and avoid contact with the rocket body, decreasing the drag coefficient. Evaluating whether this theory is correct will require further study.

These results should be treated with some skepticism due to their large uncertainty and inexact procedure used to obtain them. These will be discussed in the following section.

## Limitations

Sources of Error and Effects	Significance and Evidence	Improvements
Systematic Errors		
<p>Weather: The weather at the launch location affects the <math>p_0</math> and <math>T</math> experienced by the rocket. Changes in these variables affect the <math>c_d</math> curves produced. The weather at the launch location tends to be rather fluid, so I am unable to avoid changes in these variables.</p>	<p>Low significance. The weather was not observed to change significantly during the rocket's launches.  The weather conditions during each launch stayed relatively constant, so their effects apply evenly on</p>	<p>Launches should be done as quickly as possible to minimize changes in weather.  Another way to prevent errors from weather would be to conduct this experiment in a controlled wind tunnel.</p>

	the collected data and do not change the results of this experiment.	
<p>Air density: The air density around the rocket drops as the rocket ascends in the atmosphere. Since air density <math>\rho</math> is a factor in the <math>c_d</math> equation, changes in <math>\rho</math> affect calculated <math>c_d</math> values.</p>	<p>Low significance Air density impacts the calculated <math>c_d</math> values. Both air density <math>\rho</math> and velocity <math>u</math> decrease over time in the collected data. This means that changes in <math>c_d</math> caused by changes in <math>\rho</math> could be mistaken for a trend caused by changes in <math>u</math>.</p> <p>After doing some testing, the difference in air density between the lowest and highest <math>\rho</math> values only result in a change of around ~4% in the calculated <math>c_d</math> values, making it a rather insignificant factor compared to <math>u</math></p>	<p>It is impossible to avoid this factor if one launches a model rocket conventionally. To avoid changes in air density this experiment should be conducted in a controlled wind tunnel.</p>
<p>Smoothing height data: Noise in the <i>Pnut</i> altimeter's altitude readings were smoothed out by fitting a quadratic function to them as shown in Figure 5 and Figure 6. Values produced by the quadratic function are used for subsequent calculations.</p>	<p>Low significance The quadratic equation fits the data very well with values of <math>R^2</math> very close to 1 as shown in Figure 5 and Figure 6.</p> <p>The quadratic equation likely increased the accuracy of the investigation by eliminating noise that would have resulted in scattered velocity and acceleration data.</p> <p>The quadratic equation averaged the errors of the altitude readings, so it was not significantly affected by any single error.</p>	<p>Using an altimeter with an accelerometer would have negated the need for this.</p>
<p>Horizontal movement assumption: This investigation assumed that the horizontal distance traveled by the rocket was negligible, as most of its travel was vertical. This investigation also assumed that there was no wind. By accepting these assumptions, the vertical velocity of the rocket calculated from <math>h_a</math> can be used as <math>u</math>, the flow speed of the rocket relative to the atmosphere.</p>	<p>High Significance. Ignoring horizontal movement results in significantly underestimated velocity and acceleration values.</p> <p>Since acceleration <math>a</math> is the primary value that <math>c_d</math> calculations are based on, errors with it are especially pronounced.</p>	<p>Using an altimeter with GPS recording would alleviate this issue. Using an altimeter with an accelerometer would alleviate this issue for acceleration values. Another way to prevent this error would be to conduct the experiment in a controlled wind tunnel.</p>
<p>Humidity assumption: Humidity influences air density, which helps to determine the rocket's drag coefficient. This study will ignore the effects of humidity on the drag coefficient.</p>	<p>Insignificant. According to Niskanen (Niskanen, 2013), the effect of humidity on air density is negligible, as the difference in air density between dry air and saturated air at standard conditions is less than 1%.</p>	<p>Being insignificant, there is no improvement needed.</p>
<b>Random Errors</b>		
<p><i>Pnut</i> altimeter precision The <i>Pnut</i> altimeter has an inherent uncertainty of <math>\pm(0.1\%</math> of reading + 1 foot). Since the <i>Pnut</i>'s recorded values are used for all</p>	<p>Insignificant The smoothing process eliminated the noise created by this uncertainty.</p>	<p>Being insignificant, there is no improvement needed.</p>

subsequent calculations, it could cause errors for the rest of the investigation.		
Flowx and AccuWeather Precision The values given by these applications were assumed to have uncertainties of their last significant figure. Errors in their values could affect calculations throughout this study.	Insignificant Their uncertainties were insignificant compared to the main recorded values.	Being insignificant, there is no improvement needed.
Google Earth Precision The $h_i$ values given by Google Earth were assumed to have uncertainties of their last significant figure. Errors in its $h_i$ value could affect calculations of $h$ , $p$ , $\rho$ , and $c_d$ .	Insignificant Its uncertainty of $\pm 1$ meter are insignificant compared to the larger $h$ values used for the rest of this study.	Being insignificant, there is no improvement needed.

## Acknowledgments

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## References

- AccuWeather. (2021). *AccuWeather: Weather alerts & live forecast info* (Version 7.7.2-2-google) [Android].  
AccuWeather. <https://play.google.com/store/apps/details?id=com.accuweather.android&hl=en&gl=us>
- Enzure Digital Weather App. (2021). *Flowx: Weather Map Forecast* (3.320) [Android]. Enzure Digital Weather App. <https://play.google.com/store/apps/details?id=com.enzuredigital.weatherbomb>
- Google. (n.d.). *Google Earth*. Google Earth. Retrieved April 11, 2021, from [https://earth.google.com/web/search/36.851667,+76.746667/@36.85147978,-76.74778284,25.57432486a,126.08993005d,35y,22.81832961h,9.25002583t,0r/data=CigiJgokCW790Mzn1kJAEbQepiXE\\_UFAGUJlbkTanVLAITUTlw0RwFPA](https://earth.google.com/web/search/36.851667,+76.746667/@36.85147978,-76.74778284,25.57432486a,126.08993005d,35y,22.81832961h,9.25002583t,0r/data=CigiJgokCW790Mzn1kJAEbQepiXE_UFAGUJlbkTanVLAITUTlw0RwFPA)
- Milligan, T. V. (2012). Determining the drag coefficient of a model rocket using accelerometer based payloads. *R&D Event NARAM-54*, 17.  
[https://www.apogeerockets.com/downloads/Cd\\_determination\\_using\\_Coast\\_Data.pdf](https://www.apogeerockets.com/downloads/Cd_determination_using_Coast_Data.pdf)
- NASA. (n.d.). *Flight of a Model Rocket*. National Aeronautics and Space Administration. Retrieved April 11, 2021, from <https://www.grc.nasa.gov/WWW/k-12/rocket/rktflight.html>
- Niskanen, S. (2013). *OpenRocket technical documentation*.
- Niskanen, S., Pedrick, D., Ruland, K., Kuker, B., Graham, R., Blood, J., & du Reau, B. (2021). *Openrocket/openrocket* [Java]. openrocket. <https://github.com/openrocket/openrocket> (Original work published 2012)
- Truong, Q. (2021). *Uncertainty Calculator*. Uncertainty Calculator. <https://uncertaintycalculator.com/>

## Appendix

Table 10 Launch 1 Data

Time [s]	$h_a$ [ft]	$h_a$ [m]	$h$ [m]	smoothed [m]	$h$ $u$ [m/s]	$a$ [m s <sup>-2</sup> ]	$p$ [Pa]	$\rho$ [kg m <sup>-3</sup> ]	$F_d$ [N]	$c_d$
2.55	480	146.304	171.304	172.3658	-	-	100618	1.242325	-	-

2.6	495	150.876	175.876	175.7445	67.2761	-	100576.8	1.241816	-	-
2.65	502	153.0096	178.0096	179.0934	66.6794	-11.9344	100536.1	1.241313	-1.1478	0.121572752
2.7	513	156.3624	181.3624	182.4125	66.0827	-11.9344	100495.7	1.240814	-1.1478	0.123828007
2.75	522	159.1056	184.1056	185.7017	65.4860	-11.9344	100455.7	1.24032	-1.1478	0.126145214
2.8	537	163.6776	188.6776	188.9611	64.8892	-11.9344	100416	1.239831	-1.1478	0.128526662
2.85	548	167.0304	192.0304	192.1906	64.2925	-11.9344	100376.8	1.239346	-1.1478	0.130974746
2.9	558	170.0784	195.0784	195.3903	63.6958	-11.9344	100337.9	1.238866	-1.1478	0.133491976
2.95	567	172.8216	197.8216	198.5602	63.0991	-11.9344	100299.4	1.238391	-1.1478	0.136080978
3	577	175.8696	200.8696	201.7002	62.5023	-11.9344	100261.2	1.23792	-1.1478	0.138744507
3.05	589	179.5272	204.5272	204.8104	61.9056	-11.9344	100223.5	1.237454	-1.1478	0.141485450
3.1	601	183.1848	208.1848	207.8908	61.3089	-11.9344	100186.1	1.236992	-1.1478	0.144306836
3.15	609	185.6232	210.6232	210.9413	60.7122	-11.9344	100149.1	1.236535	-1.1478	0.147211841
3.2	620	188.976	213.976	213.9620	60.1155	-11.9344	100112.5	1.236083	-1.1478	0.150203803
3.25	631	192.3288	217.3288	216.9529	59.5187	-11.9344	100076.3	1.235636	-1.1478	0.153286226
3.3	640	195.072	220.072	219.9139	58.9220	-11.9344	100040.4	1.235193	-1.1478	0.156462792
3.35	651	198.4248	223.4248	222.8451	58.3253	-11.9344	100004.9	1.234754	-1.1478	0.159737374
3.4	662	201.7776	226.7776	225.7464	57.7286	-11.9344	99969.75	1.234321	-1.1478	0.163114046
3.45	671	204.5208	229.5208	228.6179	57.1318	-11.9344	99935	1.233892	-1.1478	0.166597094
3.5	679	206.9592	231.9592	231.4596	56.5351	-11.9344	99900.62	1.233467	-1.1478	0.170191033
3.55	689	210.0072	235.0072	234.2715	55.9384	-11.9344	99866.61	1.233047	-1.1478	0.173900619
3.6	699	213.0552	238.0552	237.0535	55.3417	-11.9344	99832.98	1.232632	-1.1478	0.177730868
3.65	709	216.1032	241.1032	239.8056	54.7450	-11.9344	99799.72	1.232221	-1.1478	0.181687067
3.7	717	218.5416	243.5416	242.5279	54.1482	-11.9344	99766.83	1.231815	-1.1478	0.185774801
3.75	725	220.98	245.98	245.2204	53.5515	-11.9344	99734.31	1.231414	-1.1478	0.189999963
3.8	733	223.4184	248.4184	247.8831	52.9548	-11.9344	99702.16	1.231017	-1.1478	0.194368782
3.85	742	226.1616	251.1616	250.5159	52.3581	-11.9344	99670.38	1.230624	-1.1478	0.198887845
3.9	751	228.9048	253.9048	253.1189	51.7613	-11.9344	99638.97	1.230237	-1.1478	0.203564120
3.95	759	231.3432	256.3432	255.6921	51.1646	-11.9344	99607.93	1.229853	-1.1478	0.208404982
4	766	233.4768	258.4768	258.2354	50.5679	-11.9344	99577.26	1.229475	-1.1478	0.213418247
4.05	774	235.9152	260.9152	260.7488	49.9712	-11.9344	99546.96	1.229101	-1.1478	0.218612197
4.1	782	238.3536	263.3536	263.2325	49.3745	-11.9344	99517.03	1.228731	-1.1478	0.223995621
4.15	792	241.4016	266.4016	265.6863	48.7777	-11.9344	99487.47	1.228366	-1.1478	0.229577845
4.2	800	243.84	268.84	268.1103	48.1810	-11.9344	99458.27	1.228006	-1.1478	0.235368779
4.25	806	245.6688	270.6688	270.5044	47.5843	-11.9344	99429.44	1.22765	-1.1478	0.241378957
4.3	814	248.1072	273.1072	272.8687	46.9876	-11.9344	99400.98	1.227298	-1.1478	0.247619587
4.35	821	250.2408	275.2408	275.2031	46.3908	-11.9344	99372.89	1.226951	-1.1478	0.254102601
4.4	829	252.6792	277.6792	277.5078	45.7941	-11.9344	99345.16	1.226609	-1.1478	0.260840715
4.45	836	254.8128	279.8128	279.7826	45.1974	-11.9344	99317.8	1.226271	-1.1478	0.267847489
4.5	843	256.9464	281.9464	282.0275	44.6007	-11.9344	99290.81	1.225938	-1.1478	0.275137396
4.55	851	259.3848	284.3848	284.2426	44.0040	-11.9344	99264.18	1.225609	-1.1478	0.282725898
4.6	858	261.5184	286.5184	286.4279	43.4072	-11.9344	99237.92	1.225285	-1.1478	0.290629526
4.65	865	263.652	288.652	288.5834	42.8105	-11.9344	99212.03	1.224965	-1.1478	0.298865970
4.7	873	266.0904	291.0904	290.7090	42.2138	-11.9344	99186.49	1.22465	-1.1478	0.307454183
4.75	880	268.224	293.224	292.8047	41.6171	-11.9344	99161.33	1.224339	-1.1478	0.316414483
4.8	885	269.748	294.748	294.8707	41.0203	-11.9344	99136.53	1.224033	-1.1478	0.325768676
4.85	893	272.1864	297.1864	296.9068	40.4236	-11.9344	99112.09	1.223731	-1.1478	0.335540193

4.9	899	274.0152	299.0152	298.9130	39.8269	-11.9344	99088.01	1.223434	-1.1478	0.345754229
4.95	905	275.844	300.844	300.8895	39.2302	-11.9344	99064.3	1.223141	-1.1478	0.356437913
5	911	277.6728	302.6728	302.8360	38.6335	-11.9344	99040.96	1.222853	-1.1478	0.367620483
5.05	917	279.5016	304.5016	304.7528	38.0367	-11.9344	99017.98	1.222569	-1.1478	0.379333490
5.1	923	281.3304	306.3304	306.6397	37.4400	-11.9344	98995.36	1.22229	-1.1478	0.391611019
5.15	928	282.8544	307.8544	308.4968	36.8433	-11.9344	98973.1	1.222015	-1.1478	0.404489935
5.2	935	284.988	309.988	310.3240	36.2466	-11.9344	98951.2	1.221745	-1.1478	0.418010165
5.25	941	286.8168	311.8168	312.1215	35.6498	-11.9344	98929.67	1.221479	-1.1478	0.432215005
5.3	947	288.6456	313.6456	313.8890	35.0531	-11.9344	98908.5	1.221218	-1.1478	0.447151467
5.35	952	290.1696	315.1696	315.6268	34.4564	-11.9344	98887.69	1.220961	-1.1478	0.462870667
5.4	958	291.9984	316.9984	317.3347	33.8597	-11.9344	98867.24	1.220708	-1.1478	0.479428267
5.45	963	293.5224	318.5224	319.0127	33.2630	-11.9344	98847.16	1.22046	-1.1478	0.496884965
5.5	968	295.0464	320.0464	320.6610	32.6662	-11.9344	98827.43	1.220217	-1.1478	0.515307054
5.55	974	296.8752	321.8752	322.2794	32.0695	-11.9344	98808.07	1.219978	-1.1478	0.534767051
5.6	979	298.3992	323.3992	323.8679	31.4728	-11.9344	98789.06	1.219743	-1.1478	0.555344411
5.65	984	299.9232	324.9232	325.4266	30.8761	-11.9344	98770.42	1.219513	-1.1478	0.577126342
5.7	988	301.1424	326.1424	326.9555	30.2793	-11.9344	98752.14	1.219287	-1.1478	0.600208728
5.75	993	302.6664	327.6664	328.4546	29.6826	-11.9344	98734.21	1.219066	-1.1478	0.624697188
5.8	998	304.1904	329.1904	329.9238	29.0859	-11.9344	98716.65	1.218849	-1.1478	0.650708284
5.85	1004	306.0192	331.0192	331.3632	28.4892	-11.9344	98699.45	1.218636	-1.1478	0.678370914
5.9	1007	306.9336	331.9336	332.7727	27.8924	-11.9344	98682.61	1.218428	-1.1478	0.707827908
5.95	1013	308.7624	333.7624	334.1524	27.2957	-11.9344	98666.12	1.218225	-1.1478	0.739237877
6	1018	310.2864	335.2864	335.5023	26.6990	-11.9344	98650	1.218026	-1.1478	0.772777354
6.05	1021	311.2008	336.2008	336.8223	26.1023	-11.9344	98634.23	1.217831	-1.1478	0.808643280
6.1	1027	313.0296	338.0296	338.1125	25.5056	-11.9344	98618.82	1.217641	-1.1478	0.847055896
6.15	1030	313.944	338.944	339.3729	24.9088	-11.9344	98603.77	1.217455	-1.1478	0.888262141
6.2	1035	315.468	340.468	340.6034	24.3121	-11.9344	98589.08	1.217274	-1.1478	0.932539623
6.25	1038	316.3824	341.3824	341.8041	23.7154	-11.9344	98574.75	1.217097	-1.1478	0.980201312
6.3	1042	317.6016	342.6016	342.9749	23.1187	-11.9344	98560.78	1.216924	-1.1478	1.031601082
6.35	1046	318.8208	343.8208	344.1159	22.5219	-11.9344	98547.16	1.216756	-1.1478	1.087140301
6.4	1050	320.04	345.04	345.2271	21.9252	-11.9344	98533.9	1.216592	-1.1478	1.147275688
6.45	1053	320.9544	345.9544	346.3085	21.3285	-11.9344	98521	1.216433	-1.1478	1.212528737
6.5	1057	322.1736	347.1736	347.3600	20.7318	-11.9344	98508.46	1.216278	-1.1478	1.283497055
6.55	1060	323.088	348.088	348.3816	20.1351	-11.9344	98496.27	1.216128	-1.1478	1.360868099
6.6	1064	324.3072	349.3072	349.3735	19.5383	-11.9344	98484.45	1.215982	-1.1478	1.445435882
6.65	1068	325.5264	350.5264	350.3355	18.9416	-11.9344	98472.98	1.21584	-1.1478	1.538121431
6.7	1071	326.4408	351.4408	351.2676	18.3449	-11.9344	98461.86	1.215703	-1.1478	1.639997966
6.75	1074	327.3552	352.3552	352.1700	17.7482	-11.9344	98451.11	1.21557	-1.1478	1.752322113
6.8	1077	328.2696	353.2696	353.0425	17.1514	-11.9344	98440.71	1.215442	-1.1478	1.876572835
6.85	1080	329.184	354.184	353.8851	16.5547	-11.9344	98430.66	1.215318	-1.1478	2.014500372
6.9	1083	330.0984	355.0984	354.6979	15.9580	-11.9344	98420.97	1.215198	-1.1478	2.168188211
6.95	1086	331.0128	356.0128	355.4809	15.3613	-11.9344	98411.64	1.215083	-1.1478	2.340132237
7	1088	331.6224	356.6224	356.2341	14.7646	-11.9344	98402.67	1.214972	-1.1478	2.533342705
7.05	1091	332.5368	357.5368	356.9574	14.1678	-11.9344	98394.05	1.214866	-1.1478	2.751476896
7.1	1094	333.4512	358.4512	357.6508	13.5711	-11.9344	98385.79	1.214764	-1.1478	2.999013475
7.15	1097	334.3656	359.3656	358.3145	12.9744	-	98377.89	1.214666	-	-



7.2 1098 334.6704 359.6704 358.9483 - - 98370.34 1.214573 - -

Table 11 Launch 2 Data

Time [s]	$h_a$ [ft]	$h_a$ [m]	$h$ [m]	smoothed $h$ [m]	$u$ [m/s]	$a$ [m s <sup>-2</sup> ]	$p$ [Pa]	$\rho$ [kg m <sup>-3</sup> ]	$F_d$ [N]	$c_d$
2.25	359	109.4232	134.4232	134.5686	-	-	101642.1	1.246137	-	-
2.3	370	112.776	137.776	138.3879	76.0539	-	101595.5	1.245564	-	-
2.35	384	117.0432	142.0432	142.174	75.3910	-13.2566	101549.2	1.244998	-1.9596	0.1618890226
2.4	396	120.7008	145.7008	145.927	74.7282	-13.2566	101503.4	1.244436	-1.9596	0.1648480053
2.45	412	125.5776	150.5776	149.6468	74.0654	-13.2566	101458	1.243879	-1.9596	0.1678868189
2.5	423	128.9304	153.9304	153.3335	73.4026	-13.2566	101413.1	1.243328	-1.9596	0.1710083597
2.55	435	132.588	157.588	156.9871	72.7397	-13.2566	101368.5	1.242782	-1.9596	0.1742156565
2.6	446	135.9408	160.9408	160.6075	72.0769	-13.2566	101324.4	1.242241	-1.9596	0.1775118780
2.65	455	138.684	163.684	164.1948	71.4141	-13.2566	101280.7	1.241706	-1.9596	0.1809003405
2.7	463	141.1224	166.1224	167.7489	70.7512	-13.2566	101237.5	1.241175	-1.9596	0.1843845166
2.75	476	145.0848	170.0848	171.2699	70.0884	-13.2566	101194.6	1.24065	-1.9596	0.1879680437
2.8	492	149.9616	174.9616	174.7578	69.4256	-13.2566	101152.2	1.24013	-1.9596	0.1916547336
2.85	500	152.4	177.4	178.2125	68.7627	-13.2566	101110.2	1.239615	-1.9596	0.1954485831
2.9	513	156.3624	181.3624	181.634	68.0999	-13.2566	101068.6	1.239105	-1.9596	0.1993537839
2.95	527	160.6296	185.6296	185.0225	67.4371	-13.2566	101027.4	1.2386	-1.9596	0.2033747350
3	540	164.592	189.592	188.3777	66.7742	-13.2566	100986.7	1.238101	-1.9596	0.2075160549
3.05	544	165.8112	190.8112	191.6999	66.1114	-13.2566	100946.4	1.237606	-1.9596	0.2117825946
3.1	555	169.164	194.164	194.9889	65.4486	-13.2566	100906.5	1.237117	-1.9596	0.2161794521
3.15	570	173.736	198.736	198.2447	64.7857	-13.2566	100867	1.236633	-1.9596	0.2207119874
3.2	581	177.0888	202.0888	201.4675	64.1229	-13.2566	100827.9	1.236154	-1.9596	0.2253858390
3.25	592	180.4416	205.4416	204.657	63.4601	-13.2566	100789.2	1.23568	-1.9596	0.2302069414
3.3	601	183.1848	208.1848	207.8135	62.7972	-13.2566	100751	1.235211	-1.9596	0.2351815440
3.35	610	185.928	210.928	210.9368	62.1344	-13.2566	100713.2	1.234747	-1.9596	0.2403162313
3.4	619	188.6712	213.6712	214.0269	61.4716	-13.2566	100675.7	1.234289	-1.9596	0.2456179446
3.45	632	192.6336	217.6336	217.0839	60.8087	-13.2566	100638.8	1.233835	-1.9596	0.2510940055
3.5	642	195.6816	220.6816	220.1078	60.1459	-13.2566	100602.2	1.233386	-1.9596	0.2567521415
3.55	654	199.3392	224.3392	223.0985	59.4831	-13.2566	100566	1.232943	-1.9596	0.2626005129
3.6	660	201.168	226.168	226.0561	58.8202	-13.2566	100530.2	1.232505	-1.9596	0.2686477422
3.65	671	204.5208	229.5208	228.9805	58.1574	-13.2566	100494.9	1.232071	-1.9596	0.2749029464
3.7	680	207.264	232.264	231.8718	57.4946	-13.2566	100460	1.231643	-1.9596	0.2813757707
3.75	690	210.312	235.312	234.73	56.8317	-13.2566	100425.5	1.23122	-1.9596	0.2880764261
3.8	700	213.36	238.36	237.555	56.1689	-13.2566	100391.4	1.230802	-1.9596	0.2950157299
3.85	707	215.4936	240.4936	240.3469	55.5061	-13.2566	100357.7	1.230389	-1.9596	0.3022051487
3.9	715	217.932	242.932	243.1056	54.8433	-13.2566	100324.4	1.229981	-1.9596	0.3096568464
3.95	724	220.6752	245.6752	245.8312	54.1804	-13.2566	100291.5	1.229578	-1.9596	0.3173837355
4	731	222.8088	247.8088	248.5237	53.5176	-13.2566	100259.1	1.22918	-1.9596	0.3253995331
4.05	739	225.2472	250.2472	251.183	52.8548	-13.2566	100227	1.228787	-1.9596	0.3337188215
4.1	748	227.9904	252.9904	253.8091	52.1919	-13.2566	100195.4	1.228399	-1.9596	0.3423571150
4.15	758	231.0384	256.0384	256.4022	51.5291	-13.2566	100164.1	1.228016	-1.9596	0.3513309322

4.2	768	234.0864	259.0864	258.962	50.8663	-13.2566	100133.3	1.227638	-1.9596	0.3606578747
4.25	775	236.22	261.22	261.4888	50.2034	-13.2566	100102.9	1.227265	-1.9596	0.3703567139
4.3	783	238.6584	263.6584	263.9824	49.5406	-13.2566	100072.9	1.226897	-1.9596	0.3804474851
4.35	791	241.0968	266.0968	266.4428	48.8778	-13.2566	100043.3	1.226535	-1.9596	0.3909515913
4.4	801	244.1448	269.1448	268.8702	48.2149	-13.2566	100014.1	1.226177	-1.9596	0.4018919169
4.45	809	246.5832	271.5832	271.2643	47.5521	-13.2566	99985.31	1.225824	-1.9596	0.4132929521
4.5	817	249.0216	274.0216	273.6254	46.8893	-13.2566	99956.93	1.225476	-1.9596	0.4251809304
4.55	823	250.8504	275.8504	275.9533	46.2264	-13.2566	99928.96	1.225133	-1.9596	0.4375839791
4.6	829	252.6792	277.6792	278.248	45.5636	-13.2566	99901.39	1.224795	-1.9596	0.4505322867
4.65	836	254.8128	279.8128	280.5096	44.9008	-13.2566	99874.23	1.224462	-1.9596	0.4640582858
4.7	844	257.2512	282.2512	282.7381	44.2379	-13.2566	99847.48	1.224134	-1.9596	0.4781968572
4.75	851	259.3848	284.3848	284.9334	43.5751	-13.2566	99821.13	1.223811	-1.9596	0.4929855543
4.8	861	262.4328	287.4328	287.0956	42.9123	-13.2566	99795.18	1.223493	-1.9596	0.5084648532
4.85	869	264.8712	289.8712	289.2246	42.2494	-13.2566	99769.64	1.22318	-1.9596	0.5246784298
4.9	875	266.7	291.7	291.3205	41.5866	-13.2566	99744.5	1.222871	-1.9596	0.5416734680
4.95	882	268.8336	293.8336	293.3833	40.9238	-13.2566	99719.77	1.222568	-1.9596	0.5595010038
5	888	270.6624	295.6624	295.4129	40.2609	-13.2566	99695.44	1.22227	-1.9596	0.5782163084
5.05	893	272.1864	297.1864	297.4094	39.5981	-13.2566	99671.51	1.221977	-1.9596	0.5978793174
5.1	898	273.7104	298.7104	299.3727	38.9353	-13.2566	99647.98	1.221688	-1.9596	0.6185551113
5.15	905	275.844	300.844	301.3029	38.2725	-13.2566	99624.86	1.221405	-1.9596	0.6403144544
5.2	912	277.9776	302.9776	303.2	37.6096	-13.2566	99602.14	1.221126	-1.9596	0.6632344012
5.25	918	279.8064	304.8064	305.0639	36.9468	-13.2566	99579.82	1.220852	-1.9596	0.6873989798
5.3	924	281.6352	306.6352	306.8947	36.2840	-13.2566	99557.91	1.220584	-1.9596	0.7128999625
5.35	930	283.464	308.464	308.6923	35.6211	-13.2566	99536.39	1.22032	-1.9596	0.7398377391
5.4	936	285.2928	310.2928	310.4568	34.9583	-13.2566	99515.28	1.220061	-1.9596	0.7683223059
5.45	941	286.8168	311.8168	312.1881	34.2955	-13.2566	99494.57	1.219807	-1.9596	0.7984743905
5.5	948	288.9504	313.9504	313.8863	33.6326	-13.2566	99474.25	1.219558	-1.9596	0.8304267333
5.55	954	290.7792	315.7792	315.5514	32.9698	-13.2566	99454.34	1.219314	-1.9596	0.8643255506
5.6	961	292.9128	317.9128	317.1833	32.3070	-13.2566	99434.83	1.219075	-1.9596	0.9003322105
5.65	966	294.4368	319.4368	318.7821	31.6441	-13.2566	99415.72	1.218841	-1.9596	0.9386251566
5.7	970	295.656	320.656	320.3477	30.9813	-13.2566	99397.01	1.218611	-1.9596	0.9794021237
5.75	974	296.8752	321.8752	321.8802	30.3185	-13.2566	99378.69	1.218387	-1.9596	1.0228826947
5.8	979	298.3992	323.3992	323.3796	29.6556	-	99360.78	1.218167	-	-
5.85	984	299.9232	324.9232	324.8458	-	-	99343.27	1.217952	-	-