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Research Article

The Physics Behind Synchronized Swimming

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ABSTRACT

In Synchronized Swimming, arguably the most demanding sport known to man, one of the most basic positions is called a vertical. In this position a swimmer's upper body is submerged in water and their legs are held above the surface while their body is kept in a straight line. Along with the buoyancy forces of the surrounding water and the air in the lungs, swimmers must also support themselves by making movements called sculls with their arms that propel them upwards. This additional force is the applied force. The goal of this research is to use physics principles to create a mathematical model that will help assist synchronized swimmers in maximizing their scores for the vertical position. The math done in this model confirmed that the amount of applied force inversely correlates with the buoyancy force needed to lift the synchronized swimmer out of the water. Additionally, the total force pushing the synchronized swimmer upwards is the same at each level. When the collected data is fitted to a second-order polynomial comparing applied sculling force to desired score, the graph shows that the data had an R^2 fit of 0.984. This knowledge could ultimately inform athletes about how to use buoyancy and other forces to their advantage which could increase their performance levels.

Objective

The overall purpose of this research is to discover how buoyancy forces and applied force correlate with the position and height of a human body submerged in water. Furthermore, the objective of this project is to create a mathematical model that can find out how much force is required to lift a synchronized swimmer out of the water to get a certain score in the vertical position.

This project will demonstrate how the buoyancy and applied forces affect the human body when submerged in water by calculating how much applied force must be added to the forces of buoyancy acting on the synchronized swimmer to attain certain scores for their height in the vertical.

Background Information

About Synchronized Swimming

Synchronized swimming is arguably one of the most difficult sports ever created. It is a water sport that combines elements of gymnastics, ballet, and swimming. Synchronized swimmers usually perform routines either as solos (where one person swims), duets (where two people swim), team routines (where four to eight people swim), and combination routines, a mix of all the other types of the routines (where eight to twelve people swim). Their routines are made up of four main components: arm work, boosts, lifts, and figures. Arm work is the intricate arm choreography that synchronized swimmers do above the surface. Boosts are the movements where a synchronized swimmer thrusts her upper body out of the water and, if executed well, gets her hips above the surface. Lifts are in routines with many people where one or multiple people are thrown out of the water and high into the air. Figures are the parts of routines where synchronized swimmers hold their breaths for lengthy periods of time while they stick their legs out of the water and make elegant movements.

In figures, the most basic position is a vertical where a synchronized swimmer is in a straight-line upside down in the water. This is the position that this project will focus on since it is one of the most basic positions. To support themselves, swimmers move their arms under the water. This is called sculling. Sculling is what propels synchronized swimmers upwards and is therefore the applied upward force in this situation. The other forces acting on the synchronized swimmer are gravity since this problem takes place on the earth which is a planet with a gravitational field, the water's buoyancy because this problem requires the synchronized swimmer to be partially submerged in water, and the force of the air in the swimmer's lungs that decrease the swimmer's overall density making them lighter than the surrounding water causing the swimmer to be pushed upward.

Synchronized Swimming Scoring Scale

Figure 1 depicts the synchronized swimming scoring scale for the vertical position. The higher a swimmer can get their body out of the water, the more points they can attain. Note that in synchronized swimming, the scoring starts from zero, which is given for not doing anything, and goes up to ten, which is given for performing an element perfectly. While height is not the only aspect that judges look for while scoring, it is the aspect that is most directly related to forces and is therefore the easiest to model mathematically.

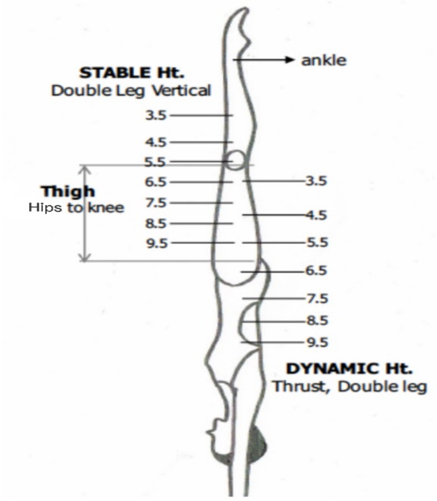

Figure 1 - Vertical Position Scoring. This diagram depicts the height a swimmer must maintain above the water in order to achieve a certain score while in the vertical position. The scoring for the vertical position is on the left side of the diagram.

The lowest level for this position is the swimmer's "natural floating level," the level where she does not need to apply any force to float. The "natural floating level" is a three-and-a-half-point vertical which is at the swimmer's mid shin. The next levels are the four-and-a-half-point vertical which is below the knee at the swimmer's upper shin, the five-and-a-half-point vertical which is at the swimmer's knee level, the six-and-a-half-point vertical which is above the kneecap at a swimmers low thigh, the seven-and-a-half-point vertical which is at the middle a swimmer's low thigh, the eight-and-a-half-point vertical which is at a swimmers mid-thigh, the nine-and-a-half-point vertical which is at the middle of a swimmers upper thigh, and the ten-point vertical which is at a swimmers hip level (not shown in Figure 1).

Physics Related to Synchronized Swimming

Physics is the math behind science and many phenomena in the real world. In this problem, Archimedes principle, a few of the most basic forces, and volume will be used to determine the amount of force a synchronized swimmer is required to exert in order to get a certain score.

Vocabulary Terms

- Force- a push or pull on an object, anything that changes an object's inertia (inertia is an object's ability to remain unchanged motion-wise)
- Newton- A unit of measurement for force named after a famous English physicist, equivalent to a kilogram meter per second squared ($kg * \frac{m}{s^2}$)
- Gravity- the force that pulls objects towards each other, on earth it accelerates objects 9.8 meters per a second squared ($9.8\,\frac{m}{s^2}$)
- Applied Force- the force applied to an object, the unbalanced force that makes an object move
- Buoyancy- the ability to float in water or other fluids, the force that pushes upwards on objects in a body of water or other fluids
- Net force- The total force acting on an object, all the forces acting on an object added together
- Free body diagram- a diagram that shows an object's net force
- Archimedes principle- a principle in physics that states that any object submerged in a fluid (a gas or liquid) is acted upon by an upward force called buoyancy that's magnitude is equal to the weight of the fluid displaced by the object
- Vertical position- a position in synchronized swimming where the swimmer is upside down and in a straight line, one of the most basic synchronized swimming positions

Hypothesis and Prediction

If a synchronized swimmer with a mass of 45.5 kg is partially submerged upside down in a body of water in the vertical position, she will need to apply more force the higher she gets out of the water because the higher she is above her "natural floating level," the more force she will have to exert in order to support the weight of her body that is no longer supported by the force of buoyancy. The relationship between this applied force and the desired score can be mathematically modeled using a regression.

The Mathematical Model

This mathematical model uses the volume of the lower body to calculate the volume of the body that is not submerged and the volume that is submerged at each height. By dividing the body like this, it is possible to calculate the force of buoyancy and the force of air. Once those forces are known, the force applied can be calculated at each height using this mathematical model.

Assumptions

As in many mathematical models and in real world situations, there are some variables that are unknown. So, to be able to solve problems in physics, we must make our best judgments and provide an educated guess. In this mathematical model there are many assumptions. They are provided in the list below.

- The acceleration from gravity is equal to 9.8 meters per a second squared ($9.8\,\frac{m}{s^2}$) because that is the acceleration from gravity on earth where this model is taking place.

- The average density of water is one kilogram per liter (1 $\frac{kg}{L}$) according to Archimedes' research about water displacement.
- The average density of the human body (when exhaling) is nine hundred eighty-five one thousandths of a kilogram per a liter (0.985 $\frac{kg}{L}$) <Loumarr, 2019>.
- Since it is difficult to calculate the density of different parts of the human body, we are going to assume that the density is consistent throughout the human body and we are going to use the average density of the human body for the density of the human body.
- The synchronized swimmer in the mathematical model will be female since most synchronized swimmers are female. Additionally, the synchronized swimmer's mass is 45.5 kilograms because we need a weight for the experimental model and 45.5 kilograms is roughly 100 pounds.
- Table 1 shows reasonable estimates for the volumes of the elevated portions of the swimmer's body based off body mass percentages found during research.

Table 1 – The volume of the non-submerged portions of the synchronized swimmer's body at different scoring levels for the vertical position <Human Body Part Weights>.

Body Part	Percentage of Body Weight	Body Weight in the Problem	Volume in the Problem
Mid shin to toes (3.5)	3.34%	1.52 kg	1.54 L
Below the knees to toes (4.5)	5.35%	2.43 kg	2.47 L
Knees to toes (5.5)	6.68%	3.04 kg	3.09 L
Above knees to toes (6.5)	9.63%	4.38 kg	4.45 L
Low thigh to toes (7.5)	10.61%	4.83 kg	4.90 L
Mid-thigh to toes	12.58%	5.72 kg	5.81 L
Upper thigh to toes	15.53%	7.06 kg	7.17 L
Hips to toes/Full legs	18.48%	8.41 kg	8.54 L

- The synchronized swimmer has air in her lungs because synchronized swimmers inhale before holding their breath underwater to increase the time they can stay underwater without needing to replenish their oxygen stores.
- The synchronized swimmer is upside down and partially submerged in the water because the vertical position requires one to be upside down with their upper body submerged in the water and we are using the vertical position for this mathematical model since it is one of the most basic positions in synchronized swimming.
- After observing many synchronized swimmers' floating levels and referring to the lowest score for the vertical position, the mid shin was found to be the average natural floating level.
- When the calculations were done without consideration of the buoyancy added from the force of air, there was an extra force of 8.33 N. The air in one's lungs decreases their overall density making them lighter than the water around them causing one to be pushed upward. Thus, the force of air is 8.33 N.

Equations

Equation 1: An object's volume

- Volume (V)- An object's volume is equal to its mass divided by its density.

$$V = \frac{m}{d}$$

Equation 2: Force of gravity on an object

- Force of gravity (F_g) - The force of gravity is equal to an object's mass multiplied by its acceleration from gravity.

$$F_g = m * g$$

Equation 3: Force of buoyancy on a submerged object

- Force of buoyancy (F_b) - Archimedes principle- The force of buoyancy is equal to the submerged volume of the object multiplied by the density of the fluid the object is submerged in multiplied by the force of gravity.

$$F_b = V_s * d_{water} * g$$

Equation 4: Calculating the submerged volume of the synchronized swimmer

  - The submerged volume of the synchronized swimmer (V_s) is equal to her total volume minus the volume that is above the surface.

$$V_s = V_{total} - (V_{total} * \%_{submerged})$$

Equation 5: Calculating the applied force

- Force from air in the lungs (F_{air}) - The force from the air in the synchronized swimmers lungs can be calculated by finding the extra force that needs to be applied at the synchronized swimmer's floating level when she is applying no extra force. This force is needed because it decreases the swimmer's overall density.

- Force applied (F_a) - The applied force is the force added to the force of buoyancy that pushes the synchronized swimmer up and out of the water. The force applied is equal to the force of gravity minus the force of buoyancy.

$$F_a = F_g - F_{air} - F_b$$

The Math and Physics

Free Body Diagram for the Situation

Figure 2 shows all the forces acting on a synchronized swimmer in a vertical position.

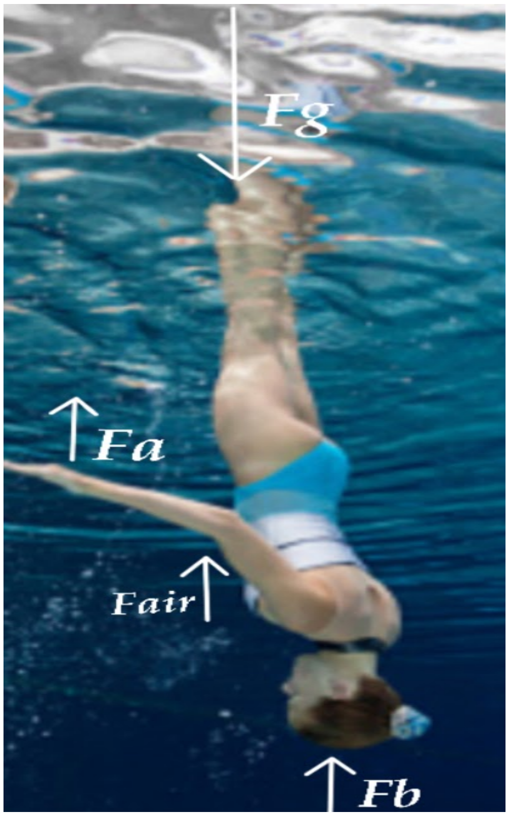

Figure 2 – Free Body Diagram showing the forces involved in the vertical position.

The force of gravity is pushing down on the swimmer; the force of buoyancy is pushing up on the swimmer; the force from air in her lungs is pushing up on the swimmer, and the applied force is pushing the swimmer up too.

Procedure

1. Calculate the force of gravity acting on the swimmer.
2. Calculate the swimmer's total volume.
3. Calculate the upward force from the air in the lungs using the "natural floating level."
4. Calculate the volume submerged at each level.
5. Calculate the force of buoyancy at each level.
6. Calculate the applied force at each level.
7. Run a regression of the necessary applied force versus the vertical score obtained to create a mathematical model.

Mathematics

The Force of Gravity

$$The\ force\ of\ gravity = the\ object's\ mass\ \cdot acceleration\ from\ gravity$$
$$F_g = m * g$$
$$F_g = 45.5\ kg * 9.8\ m/s^2$$
$$F_g = 445.9\ N$$

The Swimmer's Total Volume

$$Volume = the\ object's\ mass\ divided\ by\ its\ density$$
$$V = \frac{m}{d}$$
$$V = \frac{45.5\ kg}{0.985\frac{kg}{L}}$$
$$V = 46.19\ L$$

Force Applied at the "Natural Floating Level"/Mid Shin

Volume submerged:

$$V_s = 46.19\ L - (46.19\ L * 3.34\%)$$
$$V_s = 46.19\ L - \ 1.54\ L$$
$$V_s = 44.65\ L$$

Force of buoyancy from water:

$$F_b = V_S * d_{water} * g$$
$$F_b = 44.65\ L * 1\frac{kg}{L} * 9.8\frac{m}{s^2}$$
$$F_b = 437.57\ N$$

Force applied and Force from air in lungs:

$$F_{air} = F_g - F_b$$
$$F_{air} = 445.9\ N - 437.57\ N$$
$$F_{air} = 8.33\ N$$
$$F_a = 0\ N$$

Applied force is zero because the swimmer is not applying additional force at her natural floating level.

Force Applied Below the Knee

Volume submerged

$$V_s = 46.19\ L - (46.19\ L * 5.35\%)$$
$$V_s = 46.19\ L - 2.47\ L$$
$$V_s = 43.72\ L$$

Force of buoyancy

$$F_b = V_s * d_{water} * g$$
$$F_b = 43.72\ L * 1\frac{kg}{L} * 9.8\frac{m}{s^2}$$
$$F_b = 428.46\ N$$

Force applied

$$F_a = F_g - F_b - F_{air}$$
$$F_a = 445.9\ N - 428.46\ N - 8.33\ N$$
$$F_a = 9.11\ N$$

Force Applied at the Knee

Volume submerged

$$V_S = 46.19\ L - (46.19\ L * 6.68\%)$$
$$V_s = 46.19\ L - 3.09\ L$$
$$V_s = 43.1\ L$$

Force of buoyancy

$$F_b = V_s * d_{water} * g$$
$$F_b = 43.1\ L * 1\frac{kg}{L} * 9.8\frac{m}{s^2}$$
$$F_b = 422.38\ N$$

Force applied

$$F_a = F_g - F_b - F_{air}$$
$$F_a = 445.9\ N - 422.38\ N - 8.33\ N$$
$$F_a = 15.19N$$

Force Applied Above the Knee

Volume submerged

$$V_S = 46.19\ L - (46.19\ L * 9.63\%)$$
$$V_s = 46.19\ L - 4.45\ L$$
$$V_s = 41.74\ L$$

Force of buoyancy

$$F_b = V_s * d_{water} * g$$
$$F_b = 41.74\ L * 1\frac{kg}{L} * 9.8\frac{m}{s^2}$$
$$F_b = 409.05\ N$$

Force applied

$$F_a = F_g - F_b - F_{air}$$
$$F_a = 445.9\ N - 409.05\ N - 8.33\ N$$
$$F_a = 28.52\ N$$

Force Applied at the Low Thigh

Volume submerged

$$V_S = 46.19\ L - (46.19\ L * 10.61\%)$$
$$V_s = 46.19\ L - 4.9\ L$$
$$V_s = 41.29\ L$$

Force of buoyancy

$$F_b = V_s * d_{water} * g$$
$$F_b = 41.29\ L * 1\frac{kg}{L} * 9.8\frac{m}{s^2}$$
$$F_b = 404.64\ N$$

Force applied

$$F_a = F_g - F_b - F_{air}$$
$$F_a = 445.9\ N - 404.64\ N - 8.33\ N$$
$$F_a = 32.93\ N$$

Force Applied at the Mid-Thigh

Volume submerged

$$V_S = 46.19\ L - (46.19\ L * 12.58\%)$$
$$V_s = 46.19\ L - 5.81\ L$$
$$V_s = 40.38\ L$$

Force of buoyancy

$$F_b = V_s * d_{water} * g$$
$$F_b = 40.38\ L * 1\frac{kg}{L} * 9.8\frac{m}{s^2}$$
$$F_b = 395.72\ N$$

Force applied

$$F_a = F_g - F_b - F_{air}$$
$$\underline{F_a = 445.9\ N - 395.72\ N - 8.33\ N}$$
$$F_a = 41.85\ N$$

Force Applied at the High Thigh

Volume submerged

$$V_S = 46.19\ L - (46.19\ L * 15.53\%)$$
$$V_s = 46.19\ L - 7.17\ L$$
$$V_s = 39.02\ L$$

Force of buoyancy

$$F_b = V_s * d_{water} * g$$
$$F_b = 39.02\ L * 1\frac{kg}{L} * 9.8\frac{m}{s^2}$$

$$F_b = 382.4\ N$$

Force applied

$$F_a = F_g - F_b - F_{air}$$
$$F_a = 445.9\ N - 382.4\ N - 8.33\ N$$
$$F_a = 55.17\ N$$

Force Applied at the Hips

Volume submerged

$$V_S = 46.19\ L - (46.19\ L * 18.48\%)$$
$$V_s = 46.19\ L - 8.54\ L$$
$$V_s = 37.65\ L$$

Force of buoyancy

$$F_b = V_s * d_{water} * g$$
$$F_b = 37.65\ L * 1\frac{kg}{L} * 9.8\frac{m}{s^2}$$
$$F_b = 368.97\ N$$

Force applied

$$F_a = F_g - F_b - F_{air}$$
$$F_a = 445.9\ N - 368.97\ N - 8.33\ N$$
$$F_a = 68.6\ N$$

Results

Table 2 – Calculated applied upward forces at different scoring heights for the vertical position.

Height and Score	**Force of buoyancy + Force of air in lungs**	**Needed Force Applied**
Natural Floating Level (3.5)	445.90 N	0 N
Below the Knee (4.5)	436.79 N	9.11 N
At the Knee (5.5)	430.71 N	15.19 N
Above the Knee (6.5)	417.38 N	28.52 N
Lower Thigh (7.5)	412.97 N	32.93 N
Mid-thigh (8.5)	404.05 N	41.85 N
High thigh (9.5)	390.73 N	55.17 N
At the Hips (10)	377.3 N	68.6 N

Observations

- When the height the swimmer got out of the water increased, the force from buoyancy decreased
- When the force from buoyancy decreased, the needed force applied increased
- When the height the swimmer got out of the water increased, the need an applied force increased as well
- From the natural floating level to above the knee, the amount of needed applied force seemed to double with each height
- After the measurements for above the knee, the needed applied force increased significantly less
- For all the heights, the total amount of force pushing up on the synchronized swimmer is 445.90 N

Graph of the Results

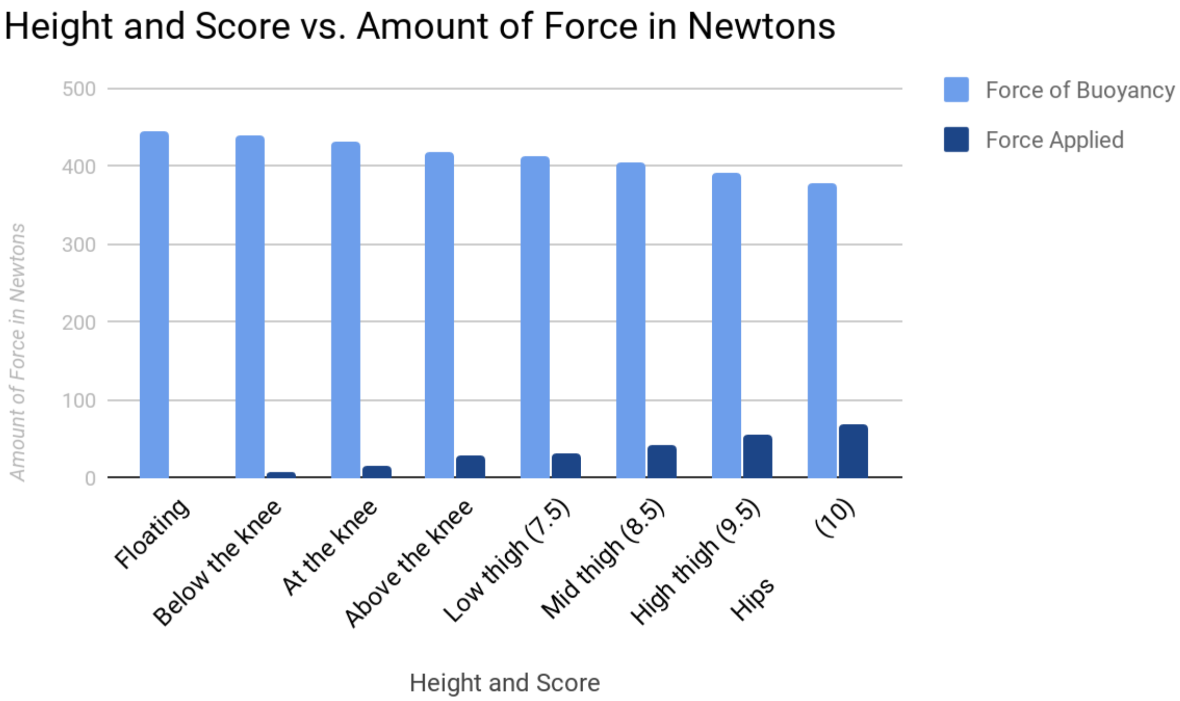

Figure 3 - Bar graph depicting changes in buoyancy and applied forces at different scoring heights.

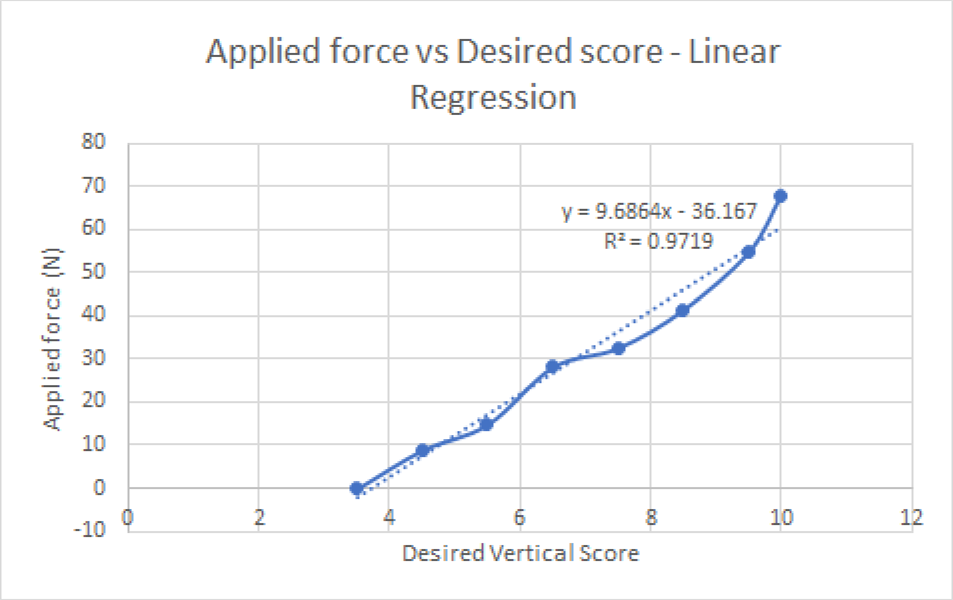

Figure 4 - Fitting a linear model

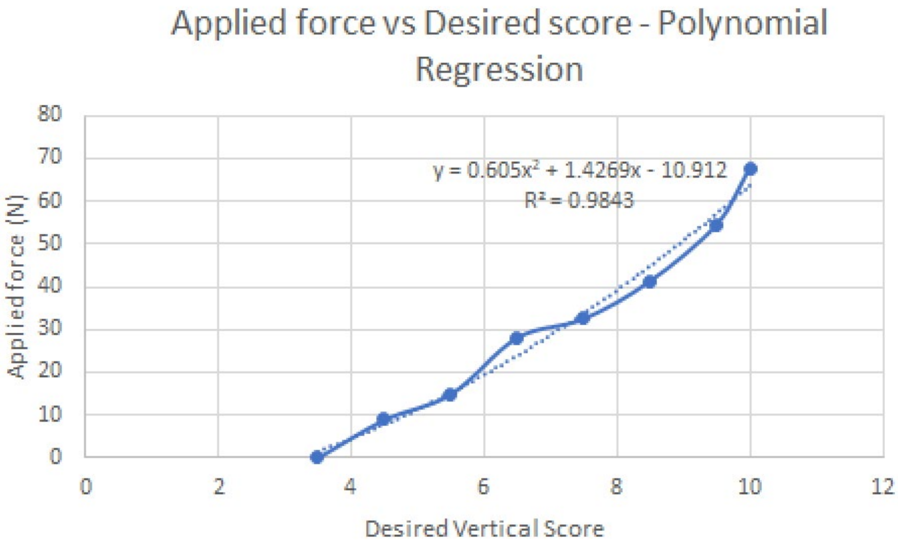

Figure 5 - Fitting a second-order polynomial model

Conclusion

Reflection of the Hypothesis

The hypothesis was, "If a synchronized swimmer with a mass of 45.5 kg is submerged upside down in a body of water, she will need to apply more force the higher she gets out of the water because the higher she is above her 'natural floating level,' the more force she will have to exert in order to support the weight of her body that is no longer supported by the force of buoyancy." This prediction is correct because as she got higher out of the water, the buoyancy force decreased so it could not support as much of her weight as it did before. Consequently, the needed applied force increased. For example, when the synchronized swimmer was her natural floating level where she would get a score of 3.5, she was applying no force and the force of buoyancy and air in her lungs was 445.9 Newtons. However, when she got higher, like at her knee level where she would get a score of 5.5, the force she applied was 15.19 Newtons which is significantly larger than zero Newtons, and the force of buoyancy combined with the force of air in her lungs was much smaller at a total of 431 Newtons. To achieve the maximum score, a swimmer of 45.5 kg will want to maintain an applied force close to 67.82 N.430.71 Newtons. The results of this experimental model make sense because as more of the body is lifted out of the water, there is less buoyancy force supporting the swimmer's weight.

Summary of the Results

The amount of force from buoyancy and the amount of applied force seemed to correlate inversely in this experimental model. When the force from buoyancy decreased as the synchronized swimmer got higher out of the water, the needed applied force to reach that height increased. Particularly, when the synchronized swimmer was at her "natural floating level," she was applying no extra force and the force of buoyancy lifted her to that height. In contrast, when the synchronized swimmer was at her hips, she was applying lots of force with a magnitude of 68.6 Newtons, and the force of buoyancy was much lower than at her natural floating level with a force of 377.3 Newtons. The most surprising trend in the data was that the values seemed to double at each height from the "natural floating level" at the swimmer's mid shin to just above the kneecap. Overall, the data gathered and the results did make sense because of Newton's second law of motion, but the fact that the values for the applied force seemed to double for the first few measurements was rather astonishing.

When fitting a mathematical model to the data, the first approach was a simple linear regression (Figure 4). The results were strong, but the model did not hold as well for higher scores. The second approach to modeling the data used a second-order polynomial and that equation fit the data much more consistently with an R^2 value of 0.9843 (Figure 5). This model yielded a final equation which can mathematically calculate the amount of applied force required for a specific desired score:

Equation 6: Mathematical model relating desired score to applied force required:

$$F_a = 0.605 * (desired\ score)^2 + 1.4269 * (desired\ score) - 10.912$$

Possible Experimental Errors

As in many experiments, this project likely has some errors in the calculations because not all of the information needed to complete this project was accessible or previously calculated and many of the calculations relied on assumptions and approximations. The place where the experimental errors were mostly likely to occur is in the measurement of the volume of the different body parts. The density and mass of each body part is extremely difficult to calculate without dismembering a human body like the one in the problem (which is probably illegal and hard to do). Therefore, there are likely some errors in this experiment since the exact statistics and information needed to complete this project are extremely hard to obtain and estimate.

Reflection of the Objective

The purpose of this project was to learn about how buoyancy and applied force correlate with the position and height of a human body submerged in water. The objective of this project was to find out how much force is required to lift a synchronized swimmer out of the water to get a certain score. This project's purpose and objective were fulfilled because there was an inverse correlation between the force of buoyancy and the applied force. Plus, the amount of force required to lift a synchronized swimmer out of the water to get a certain score was calculated. Therefore, this experimental model is very helpful in that it helps explain how Archimedes' principle, different forces, volume, and gravity relate to one another and how they work in the real world. This topic should be looked into further with more accurate measurements because it is a very intriguing topic and could be used to significantly improve synchronized swimming.

Notes for Further Experiments

One way that this project could be expanded on is by using more accurate measurements. One could also calculate the amount of energy it requires to exert such forces of energy, and furthermore, they could calculate the nutrients needed to exert such force. Another way someone could take this project to the next level is by calculating the force required to do a boost (a different component in a synchronized swimming routine) or watch a routine and calculate the total force exerted in all the different parts. Adding onto that topic, one could calculate the energy required and the nutrients needed to exert such force in a routine. The mathematical model could be expanded upon to allow calculations to be done on swimmers of different masses in addition to the 45.5 kg model used in this example. As can be seen, this project was just a gateway that leads to numerous more possible experiments. The opportunities are endless!

Real World Connection

As noted earlier, this project, and the projects it could lead to, could ultimately change synchronized swimming for the better. Experiments of this sort could help make training more effective and help athletes get the nutrients they need to thrive in their sport. This knowledge could be used and applied to many other different sports (i.e. Water polo) and in many different real-world situations.

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